Indifference prices for CO2 emission allowances

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Agenda

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2. The indifference price approach
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Kyoto protocol

- Goal: reduce Greenhouse gas (GHG) emissions
- Adopted 1997
- Target of 5% decrease in GHG emissions with respect to 1990 level for developed countries involved between 2008-2012
- Next step: Copenhagen, December 2009

The allowances system

- Each country: receives allowances for the period (2008-2012)
- End of the period: must own as much allowances as its amount of GHG emissions (1 allowance (AAU) = 1 TCO2eq)
- Flexibility: sell or buy allowances, Joint Implementation and Clean Development Mechanism

The EU ETS

- EU ETS = European Union Emission Trading Scheme. Exchange for allowances involving specific industrial sectors (power generation, cement, iron and steal, paper,...)
- First phase 2005-2007
- Second phase: Kyoto phase 2008-2012
- Phase 2: covers almost the half of the overall GHG emissions in Europe

Market rules

- Each phase: divided in yearly periods
- Beginning of the year: the state/EU decides how it distributes allowances to producers
- End of the year: each agent must own as much allowances as its yearly GHG emission amount
- In between: agent may sell/buy allowances on organized exchanges (ECX, BlueNext, EEX) or over the counter
- If excess of emissions: the agent pays a penalty (100€ per ton CO2eq)

First phase prices*

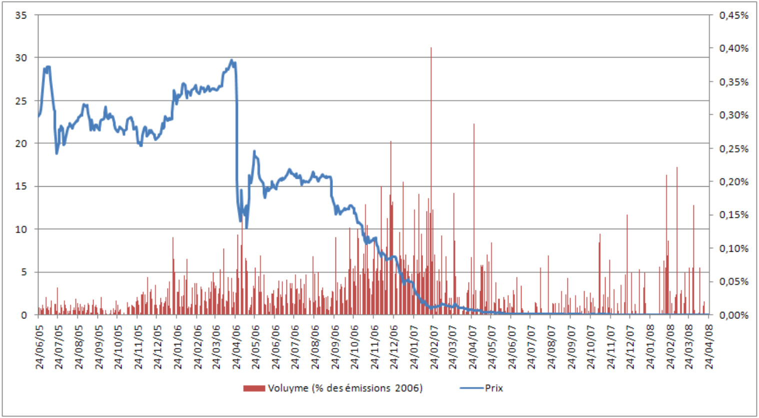

Figure 1: CO_2 Prices Phase 1

*Source: BlueNext

Indifference price: meaning of the methodology

- Individual optimization problem for an agent (producer) with respect to the CO2 market
- Captures the value beyond/below which the agent is interested in selling/buying allowances
- Deals with the risk aversion of the agent
- Prospect of understanding the sensitivity of the production behavior with respect to allowances allocation and tax design
- Not a model for market prices dynamics

Indifference price: model for an agent

- Market period: $t \in [0, T]$
- Wealth process W_t^π: stochastic process depending on time t and control π
- Criterion for optimal control process π^*:

$$\mathbb{E}\left[\mathcal{U}\left(W_T^{\pi^*}\right)\right] = \sup_\pi \mathbb{E}\left[\mathcal{U}\left(W_T^\pi\right)\right]$$

- $\mathcal{U}$ is an utility function: strictly increasing and concave. For instance an exponential utility function:

$$\mathcal{U}(x) := -e^{-\eta x}, \ \eta > 0$$

- Indifference buying price p (paid at time $t = 0$) for the asset S_T received at time T:

$$\sup_\pi \left\{\mathbb{E}\left[\mathcal{U}\left(W_T^\pi + S_T - p\right)\right]\right\} = \sup_\pi \left\{\mathbb{E}\left[\mathcal{U}\left(W_T^\pi\right)\right]\right\}$$

- For simplicity: interest rate set to zero

CO2 indifference price: elements of the model

- Random control: $(\pi_t)_{0\leq t\leq T}$: agent strategy. $\pi_t = \left(\pi_t^1, \ldots, \pi_t^n\right)'$ with $0 \leq \pi_t^i \leq p^i_{\max}$
- Random controled state variables:
  - $\mathcal{E}_t^\pi$: GHG emissions $(eT\ CO_2)$ at time t
  - W_t^π: wealth at time t
- Determinist parameters:
  - Θ_0: allocated allowances at time $t = 0$
  - $\mathcal{T}(\cdot)$: penalty function: increasing and vanishing on $\mathbb{R}_-$
  - $\mathcal{P}_{CO_2} > 0$: price for one ton CO_2 at time $t = 0$
  - q_{CO_2}: bought quantity of allowances at time $t = 0$

CO2 indifference price: definition

We define the following value functions:

$$V(t,w,e) = \max_{\pi^t} \mathbb{E}_{\{W_t=w,\mathcal{E}_t=e\}} \left[\mathcal{U}\left(W_T^{\pi^t} - \mathcal{T}\left(\mathcal{E}_T^{\pi^t} - \Theta_0 \right) \right) \right]$$

$$V_{\mathcal{P}_{CO_2}}(t,x,e,q_{CO_2}) =$$
$$\max_{\pi^t} \mathbb{E}_{\{W_t=w,\mathcal{E}_t=e\}} \left[\mathcal{U}\left(W_T^{\pi^t} - q_{CO_2}\mathcal{P}_{CO_2} - \mathcal{T}\left(\mathcal{E}_T^{\pi^t} - \Theta_0 - q_{CO_2} \right) \right) \right]$$

Indifference price for an amount q_{CO_2} of allowances: the price $\mathcal{P}^*_{CO2}$ which equalizes:

$$\boxed{V_{\mathcal{P}^*_{CO_2}}(0,x,e,q_{CO_2}) = V(0,x,e)}$$

CO2 indifference price: exponential utility case

With an exponential utility function $\mathcal{U}(x) = -e^{-\rho x}$ the indifference price is:

$$\boxed{\mathcal{P}(q) = -\frac{1}{\rho q}\log\left(\frac{v(0, w, e-q, s)}{v(0, w, e, s)}\right)}$$

No need to compute v^{-1}

CO2 indifference price: to be studied

- What sensitivity to the design:

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CO2 indifference price: to be studied

- What sensitivity to the design:
  - of the penalty $\mathcal{T}$?

CO2 indifference price: to be studied

- What sensitivity to the design:
  - of the penalty $\mathcal{T}$?
  - of the initial allocation Θ_0 ?

Why electricity ?

- Huge part of the market: 70% of the phase 1 allocation plan.
- More flexibility for emissions: fuel switching
- Inelastic demand: we represent demand with market price

Powernext DayAhead prices from January 1st, 2006 to February 5, 2009

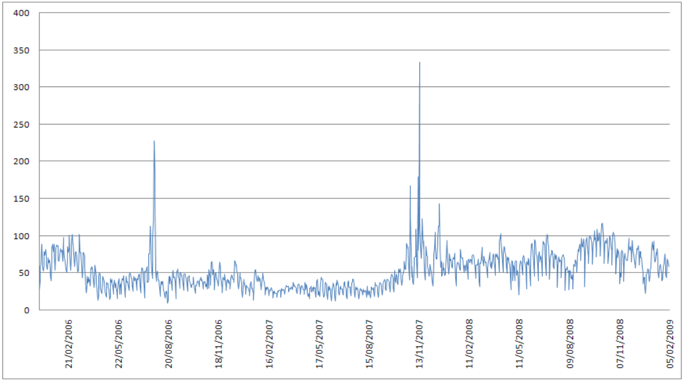

Figure 2: Electricity spot prices

The model: Wealth dynamic for the electricity producer

- The control $\pi = \left(\pi_t^1, \ldots, \pi_t^n\right)$: generated power, different plants (coal, gas, fuel,...)
- We specify the dynamic of the wealth process:

$$dW_t^\pi = \left\{ \left(\pi_t \cdot \mathbb{1} - Q_t^{OTC}\right) S_t + Q_t^{OTC} \mathcal{P}(t) - C(t, \pi_t) \right\} dt$$
$$= h(t, S_t; \pi_t)\, dt$$

- Production costs: $C(t, \pi_t)$, bounded
- Electricity spot price : S_t
- Quantity and price for contractual production: Q_t^{OTC} and $\mathcal{P}(t)$, bounded

The model: electricity Spot price and emissions dynamics

- Stochastic process for electricity spot market price

$$\begin{cases} dS_r^{t,s} = b\left(r, S_r^{t,s}\right) dr + \sigma\left(r, S_r^{t,s}\right) dB_r, \ \forall r \geq t \\ S_t^{t,s} = s \end{cases}$$

- b and σ lipschitz in s uniformly in t
- Dynamic for CO2 emissions:

$$d\mathcal{E}_t^{\pi} = \alpha(\pi)\, dt$$

- α bounded

The HJB equation associated with the stochastic control problem is:

$$\begin{cases} \dfrac{\partial v}{\partial t} + b\left(t,s\right)\dfrac{\partial v}{\partial s} + \dfrac{\sigma\left(t,s\right)^2}{2}\dfrac{\partial^2 v}{\partial s^2} \\ \quad + \sup_{\pi \in \mathcal{A}} \left\{ h\left(t,s;\pi\right)\dfrac{\partial v}{\partial w} + \alpha\left(\pi\right)\dfrac{\partial v}{\partial e} \right\} = 0 \\ \\ v\left(T,w,e,s\right) = \mathcal{U}\left(w - \mathcal{T}\left(e - \Theta_0\right)\right) \end{cases}$$

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Definition of the value function: only locally bounded, not smooth. But we still have:

Theorem

With above assumptions the value function of the stochastic control problem is a viscosity solution to the HJB equation

HJB: Uniqueness theorem

Theorem

If the functions b and σ verify the following growing conditions:

$$\exists \kappa_1,\ |b(t,s)| \leq \kappa_1 (1+s)\,.\ \forall s > 0$$
$$\exists \kappa_2,\ |\sigma(t,s)| \leq \kappa_2 \left(1+\sqrt{s}\right).\ \forall s > 0$$

and if u and v are viscosity super- and sub-solutions to the HJB equation with

$$u(T,\cdot,\cdot,\cdot) \leq v(T,\cdot,\cdot,\cdot) \text{ and } u(t,w,e,0) \leq v(t,w,e,0)$$

then the comparison holds on the entire domain:

$$u(t,w,e,s) \leq v(t,w,e,s)\,,\ \forall (t,w,e,s) \in [0,T] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_+$$

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HJB: Dimension reduction

- We consider the case of an exponential utility function: $\mathcal{U}(x) = -e^{-\rho x}$
- Then the value function v satisfies: $v(t, w, e, s) = \mathcal{U}(w)\, g(t, e, s)$
- And g satisfies the following HJB equation:

$$\begin{cases} g_t + \mathcal{G}\left(t, e, s, g, Dg, D^2 g\right) = 0 \\ g(T, x) = exp\left(\rho \mathcal{T}\left(e - \Theta_0\right)\right) \end{cases}$$

Barlow model: a model for electricity spot prices

- Model due to M.T. Barlow: Non Linear Ornstein-Uhlenbeck
- Specificity of the model: able to produce characteristic spikes of electricity spot price
- Formulation of the model:

$$\begin{cases} S_t = \begin{cases} f_\alpha(X_t), & 1+\alpha X_t > \epsilon_0, \\ \epsilon_0^{\frac{1}{\alpha}}, & 1+\alpha X_t \leq \epsilon_0 \end{cases} \\ f_\alpha(x) = (1+\alpha x)^{\frac{1}{\alpha}}, \ \alpha < 0 \\ dX_t = -\lambda (X_t - a)\, dt + \sigma dB_t \end{cases}$$

Barlow model: simulation

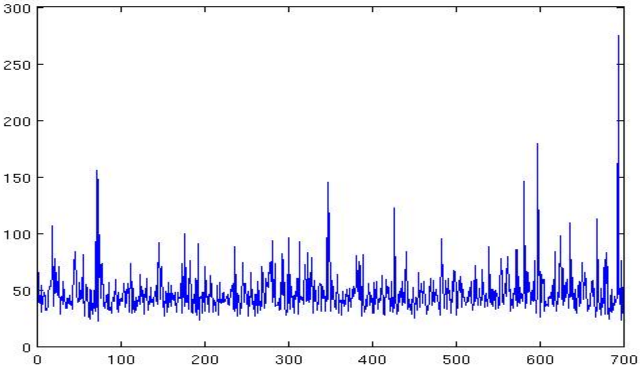

Figure 3: Barlow process simulation

Barlow model: HJB equation

With this model for the diffusion S_t we can rewrite the HJB equation in terms of the underlying diffusion X_t:

$$\frac{\partial g}{\partial t} - \lambda (x - a) \frac{\partial g}{\partial x} + \frac{\sigma^2}{2} \frac{\partial^2 g}{\partial x^2} + \inf_{\pi \in \mathcal{A}} \left\{ \alpha(\pi) \frac{\partial g}{\partial e} - \rho h(t, f_\alpha(x); \pi) g \right\} = 0$$

g can be computed via finite difference methods and thus the value v and the indifference price $\mathcal{P}$

Conclusion

- Done work: modeling, well formulated
- Work in progress: numerics, studying indifference price sensitivity (backward stochastic differential equations)