Marginal costs and Optimality for cogeneration systems

Marc Bordier, Nadia Maïzi

Centre for Applied Mathematics
MINES ParisTech

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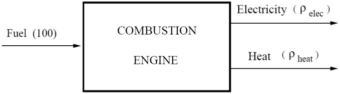
Supplying Power versus supplying Heat and Power

Cogeneration plants produce heat and power as well.



How to supply heat and electricity demand at lowest cost?

- Well known response when supplying solely electricity power :
  ☞ *selection according to merit order*

- With combined heat and power plant (CHP or cogeneration units) :
  ☞ *definition of merit order is no longer simple*

Industrial problem formulation

Given an operator supplying heat and power and wishing to solve its generation units commitment :

1. supply local heat inflexible demand
   ☞ (industrial process heat, district heating, . . .)
   - with traditional boilers
   - with CHP
2. supply electricity demand
   - locally with CHP
   - through the grid on the electricity market

Supplying Heat and Power : some remarks

1. Why co-generation ?
    - production of both power and heat
    - more efficiency than traditional (separate) production systems
    - lower environmental impacts
2. Optimizing a combined production of heat and power is not easy:
    - as there is competition between production of heat and power
    - it cannot be seen as a classical network operation for heat must be produced locally, while it is indifferent for the power

Optimal problem formulation : competitiveness

Given a station load with multiple power plants
(including cogeneration units and traditional boilers)

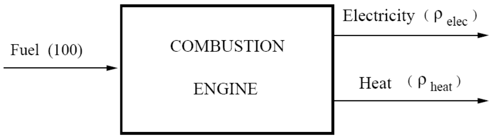
- identify the unit optimal loadings
- that minimize the total generation operation cost

☞ Is merit order scheduling still valid?

☞ Interpertation of that new merit order definition ?

A single CHP unit

Combustion engine model



Combustion engines

- wide range of electric operating modulations (or modes) m
- total efficiency ρ non-sensitive to mode variations

$$\rho = \rho_{\text{heat}}(m) + \rho_{\text{elec}}(m)$$

- affine relation between electricity and heat productions

$$\alpha\, P_{\text{elec}}(m) + \beta\, P_{\text{heat}}(m) = \text{constant}$$

Co-generation efficiency or "boiler equivalent" methodology

For a given modulation m of the combustion engine

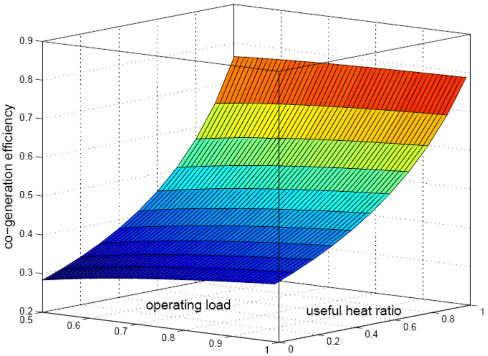
- heat production $P_{\text{heat}}(m)$
- useful heat production $\tau\, P_{\text{heat}}(m) \quad (0 \le \tau \le 1)$ (for which demand is supplied)
- given the best boiler efficiency ρ_{boiler}
- the fuel energy *savings* ratio is

$$\tau\, P_{\text{heat}}(m) / \rho_{\text{boiler}}$$

Equivalent electrical efficiency ρ_{equiv} (or co-generation efficiency)

$$\frac{1}{\rho_{\text{equiv}}(m,\tau)} = \frac{1}{\rho_{\text{elec}}(m)} - \left(\frac{\tau}{\rho_{\text{boiler}}}\right) \frac{1}{\rho_{\text{heat}}(m)}$$

ρ_{equiv} co-generation efficiency (m operating modulations, τ useful heat ratio)



$\rho_{\text{equiv}}(m, \tau)$, the equivalent electrical efficiency is an increasing function of fuel energy savings.

Operating cost for a single unit

Per unit costs:

- C_{fuel} fuel cost
- C_{omf} fixed operation and maintenance costs (do not depend on the useful energy)
- $C_v(m,\tau) = \frac{C_{\text{fuel}}}{\rho_{\text{equiv}}(m,\tau)}$ variable costs (functions of the generation level)
- C_{export} a transportation cost in case of excess electricity to be sent to the grid

Production cost per unit

$$C(m,\tau) = C_v(m,\tau) + C_{\text{omf}} + C_{\text{export}}$$

Maximum avoided cost of heat

- Let
  - $P_{\text{heat}}^{\text{demand}}$ the heat demand
  - $P_{\text{heat}}(m)$ the heat production at modulation m
- Heat ratio corresponding to maximum avoided cost of heat

$$\tau^{\star}(m) = \min\left(\frac{P_{\text{heat}}^{\text{demand}}}{P_{\text{heat}}(m)}, 1\right)$$

- Co-generation efficiency $\rho_{\text{equiv}}(m, \tau)$ is an increasing function of τ
- Best co-generation efficiency $\rho^{\star}$

$$\rho^{\star}(m) = \rho_{\text{equiv}}(m, \tau^{\star}(m))$$

Total Operating Cost Function

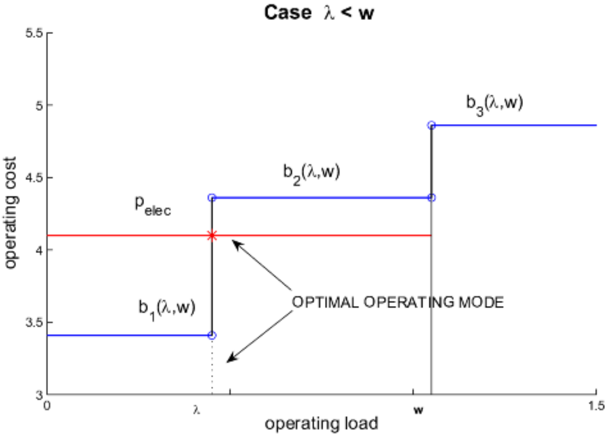
- When operating at best co-generation efficiency $\rho^\star$
- Defining the characteristics modulations λ and w :
  - λ mode for which the heat demand is satisfied $(0 \leq \lambda \leq \infty)$
  - w mode for which the electricity demand is satisfied $(0 \leq w \leq \infty)$
- Defining the electricity market price p_{elec}
- The operating cost depends on the relative positions of m, w, λ :

$$C(m, \lambda, w) = p_{\text{elec}} \max(0, w - m) + \max_{i=1,\ldots,3} \left(a_i(\lambda, w) + b_i(\lambda, w)\, m \right)$$

where $b_i(\lambda, w)$ $(i = 1, 2, 3)$ is a positive increasing sequence

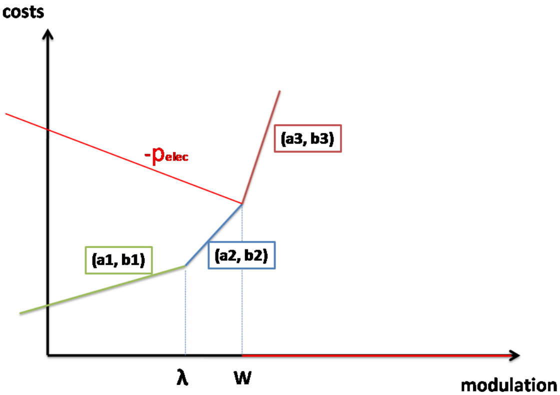
Total Operating Cost Function

It relies on the decomposition over the (a_i, b_i) coefficients



Total Operating Cost Function: decomposition

Non differentiable convex objective function



Total Operating Cost Function: decomposition

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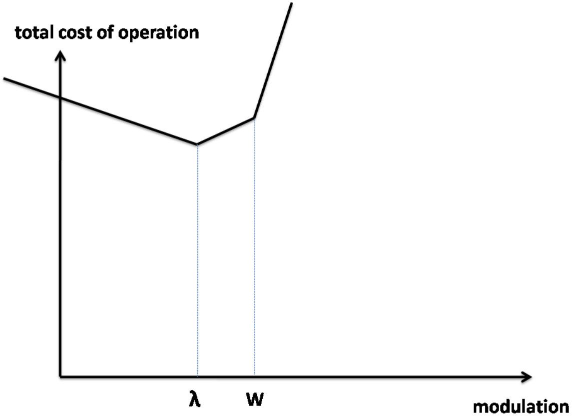


Optimal Operating Mode (1)

For given λ and w solve the minimization problem

$$\min_m C(m, \lambda, w) \qquad \text{subject to} \qquad m \in [0, 1]$$

- the operating cost function $C(m, \lambda, w)$ is convex as sum of convex functions of m but non-differentiable
- its subdifferential $\partial C(m, \lambda, w)$ wrt. m is easily obtained, by example:

$$\text{for} \quad \lambda < w \qquad \partial C(m, \lambda, w) = \begin{cases} b_1(\lambda, w) - p_{\text{elec}} & 0 \le m < \lambda \\ [b_1(\lambda, w), b_2(\lambda, w)] - p_{\text{elec}} & m = \lambda \\ b_2(\lambda, w) - p_{\text{elec}} & \lambda < m < w \\ [b_2(\lambda, w) - p_{\text{elec}}, b_3(\lambda, w)] & m = w \\ b_3(\lambda, w) & w < m \end{cases}$$

Optimal Operating Mode (2)

- The optimality condition in $m^\star$ iff

$$\exists \xi \in \mathbb{R} \quad \xi \in \partial C(m^\star, \lambda, w) \qquad \xi.(m - m^\star) \geq 0 \quad \forall m \in [0, 1]$$

yields the optimal operating mode to be one the following values

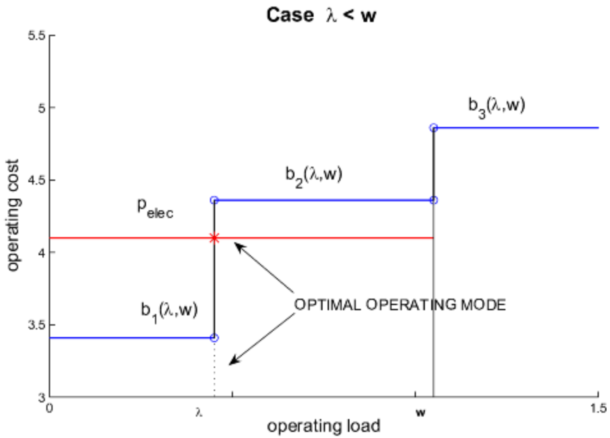
$$m^\star \in \{0 \ , \ \lambda \ , \ w \ , \ 1\}$$

according to

- the position of p_{elec} wrt. to the $b_i(\lambda, w) \quad (i = 1, 2, 3)$
- the relative positions of λ and w wrt. 0 and 1

Optimal Operating Mode (3)

Interpretation of the optimum condition



Optimal Operating Mode (end)

- The $b_i(\lambda, w) \quad (i = 1, 2, 3)$ depend on
  - the thermodynamic performance of the cogeneration system
  - the fuel cost and the fixed operation and maintenance costs
- They are interpreted as the marginal operating costs associated to the

  heat-match mode, electricity-match mode, mixed-match mode

- Conclusion:

For a single cogeneration unit

optimal operating mode : a merit order scheduling

Distributed CHP units

What's new?

Given a set of distributed cogeneration operated by a unique producer

- heat productions are still local
- electricity can be <u>exchanged</u> between units through the network

Total operating cost function

Let us define

- for unit i $\quad i = 1, 2, \ldots, N$
  - m_i the local operating mode
  - α_i the relative installed power capacity weight wrt. the overall installed power capacity
  - λ_i the mode for which the local heat demand is satisfied
  - w_i the mode for which the local electricity demand is satisfied
- for the overall units the total electricity demand is $\quad W = \sum_i \alpha_i w_i$

The total operating cost function is now

$$C(m_1, \ldots, m_N) = p_{\text{elec}} \max\left(0, W - \sum_i \alpha_i m_i\right) + \sum_i \alpha_i C_i(m_i, \lambda_i, w_i)$$

Optimal operating modes

The optimization problem reads

$$\min_{(m_1,\ldots m_N)} C(m_1,\ldots,m_N) \qquad \text{subject to} \qquad (m_1,\ldots .m_N) \in [0,1]^N$$

- The cost function is convex but non-differentiable
- The subdifferential can be obtained explicitely through
  - the Moreau-Rockafellar theorem (sum of subdifferentials)
  - the Duboviskii-Milyutin theorem (subdifferential of the supremum of a finite familly of convex functions)
- an explicit formulation of the solution can be derived

For a set of cogeneration units

optimal operating modes : a merit order dispatch

Conclusion

Summary

Combination of heat and power makes the operating mode schedule a difficult task as

- heat and power generation compete through CHP units (combustion engine)
- heat is generated locally while power is flowing through the grid

In the short and medium term, the place of CHP (among the overall generating plants) will derive from

- the structure of the subsidies for which CHP may be eligible
- the price of heat
- the structure of grid charges
- the price of gaz

1. For one CHP
    - the cost function is convex and non differentiable
    - the minimum is trivial
    - can be interpreted in terms of operating marginal costs

2. For a distributed set of CHP plants :
    - the cost function is still convex and non differentiable
    - a numerical solution can easily be derived

   but

    - a subdifferential approach yields an analytical solution
    - with an economical interpretation by means of marginal costs

optimal operating mode : a merit order scheduling