How to predict the probability of a major nuclear accident after Fukushima Dai-ichi?

François Lévêque and Lina Escobar

CERNA Mines ParisTech

March 14, 2012

Outline

Motivations

- Who is wrong? Discrepancy between observed and estimated frequency of major nuclear accidents (Core damage frequency CDF, Large Early Release Frequency LERF)

  CDF: Observed frequency is $\frac{11}{14.400}$=7.6E-04/r.y versus the estimated 1.0E-04/r.y

  LERF: Observed frequency is $\frac{4}{14.400}$=2.8E-04/r.y versus the estimated 1.0E-06/r.y

- How to combine observations and other pieces of knowledge (e.g: probabilistic safety assessments, data of safety improvements) to predict the probability of a major accident?
- How does the probability of major accident change due to the Fukushima Dai-ichi event?

Binomial distribution

The first step to compute the probability is to assume a model of accident occurrence.

Assumption 1

Let Y be the number of nuclear accidents. We are going to assume that they come from a binomial distribution.

$$f(y) \sim B(k, p)$$

$$P(Y = k) = C_n^k p^k (1-p)^{n-k}$$

Where:

- n : Number of trials = Years* Number of reactors
- k: Number of "success" = accidents
- p: Rate in which "successes" arrive= frequency

Results

Table: Binomial distribution results

	Number of reactors	1-P(k=0)
Worldwide	433	0.28
Europe	143	0.10
France	58	0.04

If we use the CDF goal (p=1.0E-0.4) we find that the probability of an accident for the next year should be 0.00043 worldwide. How can we explain such a difference?

Uncertainty in the parameter

- In the previous computations, we have assumed that the rate in which the accidents arrive, given by p equals the observed frequency. So the probability is only determined by what we have observed so far $\Rightarrow$ Frequentist approach
- An alternative methodology consists in assuming that p is a random variable, to represent the fact that our parameter is uncertain.

Bayesian inference

The Bayesian approach relies upon Bayes' law to make consistent inferences about the plausibility of an hypothesis given new information. We have three main ingredients:

1. The prior distribution: Encodes the information about the parameter's state of knowledge
2. Observations: Allow to update the hypothesis that we have made in the prior
3. Bayes' law

Let's recall Bayes' Law

$$p(H|e)\frac{p(H)*p(e|H)}{p(e)}$$

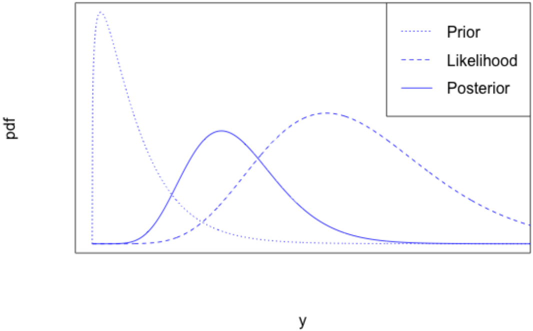
Where:

- H: Hypothesis
- e: Evidence or data

Bayesian updating

Under this approach, we take the Prior distribution for our parameter $\pi_0(p)$, and then we update it with the available data y using Bayes' law.

$$\overbrace{\pi_1(p|y)}^{\textit{Posterior}} \propto \overbrace{f(y|p)}^{\textit{Likelihood}}\ \overbrace{\pi_0(p)}^{\textit{Prior}}$$



Binomial-Beta Model

Some intuition about the Bayesian update with the binomial distribution. If the initial rate at which accidents arrive is given by:

$$p_0 = \frac{k}{n}$$

Where:

- k: Number of successes
- n: Number of trials

How does p_0 change if we add 2 more trials and we get 1 success?

$$p_1 = \frac{k+1}{n+2}$$

So if instead of 2, we get a proportion of t successes in s trials, we have:

$$p_1 = \frac{k+st}{n+s}$$

Binomial-Beta Model

Taking into account the previous slide s and t represent our initial hypothesis about nuclear accidents.

- t will represent the expected rate
- s is the strength of our prior

This information will be encoded in the conjugate distribution of the binomial $\Rightarrow$ Beta distribution

Assumption 2

We are going to assume that p follows a Beta distribution with parameters $st, (1-t)s$

$$\pi_0(p) = B\left[st, s(1-t)\right]$$

Where:

- $\mathrm{E}(p) = t$
- $\mathrm{Var}(p) = t(1-t)/(s+1)$

Binomial-Beta Model

Result

Given that the Beta distribution is the Binomial conjugate, the posterior distribution $\pi_1(p)$ is also a Beta distribution and the parameters are updated following a simple formula:

$$\pi_1(p|y_1) = B\left[\alpha_1, \beta_1\right]$$

Where:

- $\alpha_1 = st + y_1$
- $\beta_1 = s(1-t) + n - y_1$

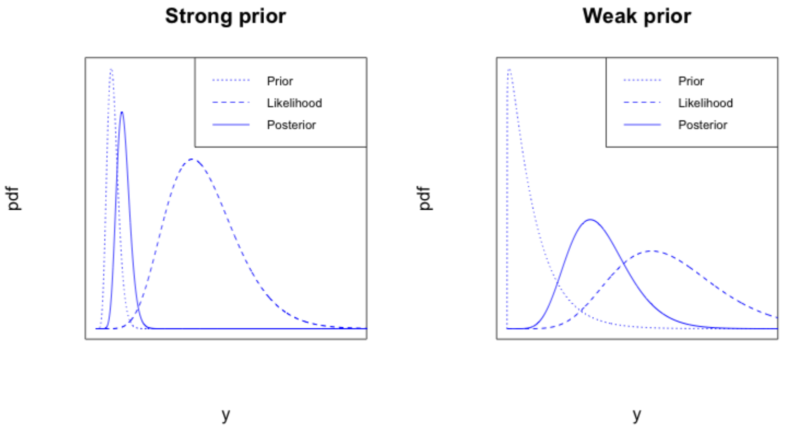
Using the Beta properties we find that:

$$\mathrm{E}(p|y_1) = \frac{y_1 + st}{n + s}$$

Does it look familiar? Yes is the same result when adding st virtual succesess in s trials.

The prior distribution

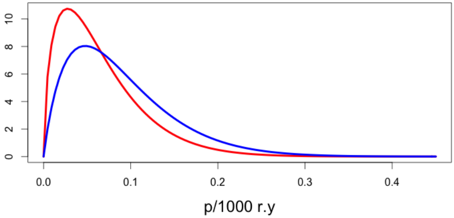
How much do the results depend on the assumed prior?



PSA results to construct our Beta prior

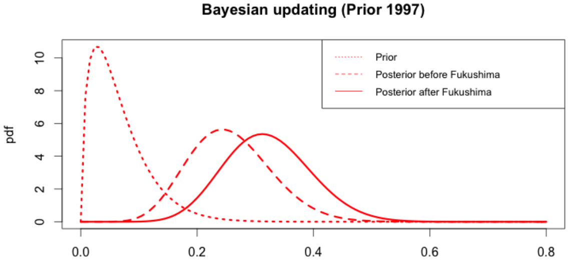
Which values of t and s we have to use in $\pi_0(p)$?. We are going to use the PSA core damage frequency results. The estimated CDF is going to be t and using the 95% quantile we can recover the s.

- The NUREG 1150 (1990) has a average CDF equal to 8,9E10-5
- The NUREG 1560 (1997) has a average CDF equal to 6,5E10-5



PSA	t	s
— NUREG 1150	8,9E10-5	21882
— NUREG 1560	6,5E10-5	24869

Bayesian results



	$E(p)$
Prior	6.50E10-5
Before Fukushima	2.56E10-4
Posterior Fukushima	3.21E10-4
Fukushima effect $\Delta\%$	0.2539

The results indicate that Fukushima Dai-ichi accident have updated the expected frequency of a nuclear accident by 25%

Assumptions underlying the probability distribution

If we compute the probability of at least one core damage in the following next year we find that:

	$1 - P(k = 0)$		
	Worldwide	Europe	France
Binomial	0.28	0.100	0.040
Bayesian	0.12	0.044	0.018

These results depend on strong assumptions underlying the models.

1. The nuclear fleet is assumed to remain constant
2. There has not been any safety progress during the development of nuclear power
3. The distribution assumes that the events are independent
4. All the reactors are the same: (i.e.,same probability regardless age, technology, localization)

Assumptions underlying the probability distribution

If we compute the probability of at least one core damage in the following next year we find that:

	$1 - P(k = 0)$		
	Worldwide	Europe	France
Binomial	0.28	0.100	0.040
Bayesian	0.12	0.044	0.018

These results depend on strong assumptions underlying the models.

1. The nuclear fleet is assumed to remain constant
2. There has not been any safety progress during the development of nuclear power
3. The distribution assumes that the events are independent
4. All the reactors are the same: (i.e.,same probability regardless age, technology, localization)

U.S PSA results

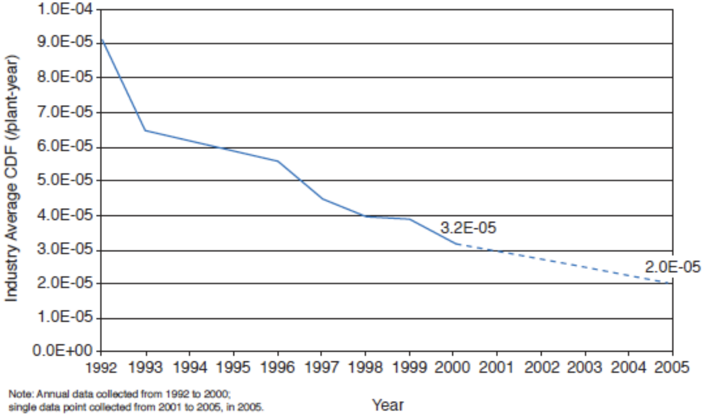


Figure: Core Damage Frequency Industry Average Trend. EPRI(2008)

Individual PSA results

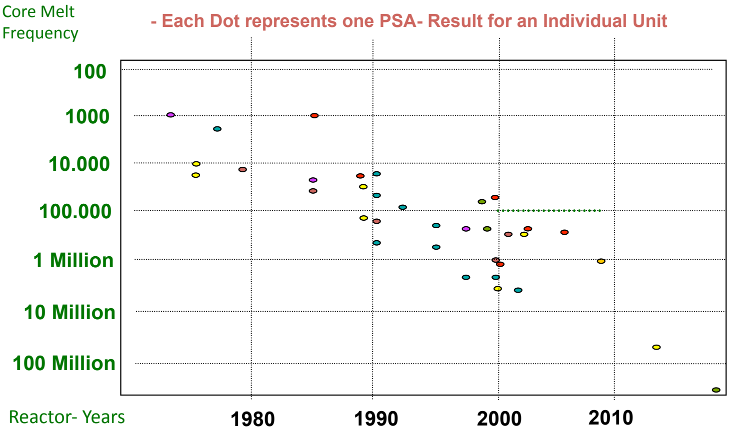
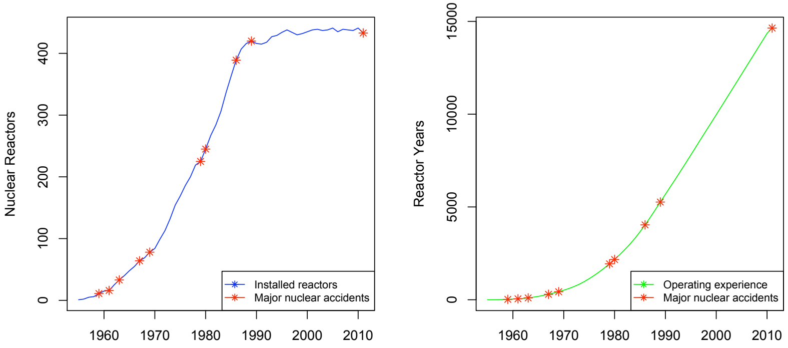


Figure: Core Damage Frequency. Mohrbach(2011)

Safety improvements



Main feature:

- Most of the accidents were observed during the first years. When nuclear industry cumulated experience exponentially only Fukushima has been observed.

Poisson regression

Assumption 1

Let y_t be the number of accident observed at time t. We are going to assume that they drawn from a Poisson distribution

$$f(y_t|\lambda) = \frac{\exp(-\lambda E_t)(-\lambda E_t)^{y_t}}{y_t!}$$

Where:

- λ is the rate in which accidents arrive (accidents per reactor year)
- E_t is the exposure time at year t, corresponds to the number of operative reactors in each year

Assumption 2

We are going to assume that the arrival rate is defined as a log-linear link function of the unexpected unavailability factor (UUF)

$$\lambda = \frac{\exp(X_t'\beta)}{E_t}$$

Poisson regression with UUF

- We choose the average Unplanned Unavailability Factor (UUF) as the explanatory variable
- It is the ratio between the amount of energy loss due to unplanned events in the plant with respect to the energy that the reference unit power could have produce during the same period

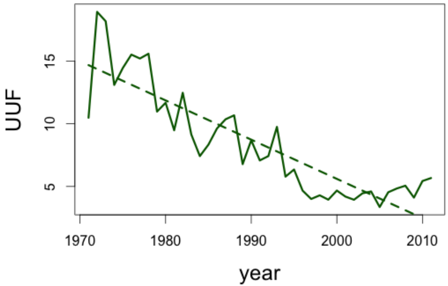


Table: Poisson with UUF

	Coefficients	Estimate Std.	Error z value	Pr(> |z|)	
(Intercept)	-8.87689	0.68970	-12.8706	<2e-16	***
uuf	0.11455	0.06257	1.8308	0.06713	.

Event count time series model

- When we assume that we have an independent and identically distributed (i.i.d) sample, we give the same weight to each observation $\Rightarrow$ In the Poisson regression all the accidents are equally important in the estimation
- But we have a time series, thus if events that we observe today are somehow correlated with those in the past $\Rightarrow$ We should give more weight to recent events than those in the past.
- We propose to use a structural event-count time series model. This framework has been developed by Harvey and Fernandes (1989) and Brandt and Williams (1998)
- This model is called Poisson Exponentially Weighted Moving Average (PEWMA)

PEWMA model

PEWMA model has a time changing mean λ_t that is described by two components:

Observed component: Given by a log-link as in Poisson regression that contains the explanatory variables that we observe at time t. We are interested in knowing how these variables affect the current state, which are represented with β coefficients

Unobserved component: Shows how shocks persist in the series, therefore it captures data dependence across time. This dependence is represented by a smoothing parameter defined as ω

ω is the key parameter of PEWMA model, because it represents how we discount past observations in current state.

- If $\omega \to 0$ this means that the shocks persist in the series. We have high dependence in the data
- If $\omega \to 1$ this will indicate that events are independent

▸ PEWMA Equations

Bayesian approach in PEWMA model

To find the density of the parameter across time we use a Kalman filter that recursively uses Bayesian updating. The procedure consists in combining a prior distribution $\Gamma(a_{t-1}, b_{t-1})$ with the transition equation to find $\pi(\lambda_t | Y_{t-1})$ that is a Gamma distribution

$$\lambda_t | Y_{t-1} \sim \Gamma(a_{t|t-1}, b_{t|t-1})$$

Where:

- $a_{t|t-1} = \omega a_{t-1}$
- $b_{t|t-1} = \omega b_{t-1} \frac{\exp(-X_t' \beta - r_t)}{E_t}$

Bayesian approach in PEWMA model

Since the Gamma is the conjugate distribution for a Poisson likelihood, we use Bayes' law and the posterior distribution is also Gamma.

$$\pi(\lambda_t|Y_t) \propto \underbrace{f(y_t|\lambda_t)}_{Poisson} \underbrace{\pi(\lambda_t|Y_{t-1})}_{Gamma}$$

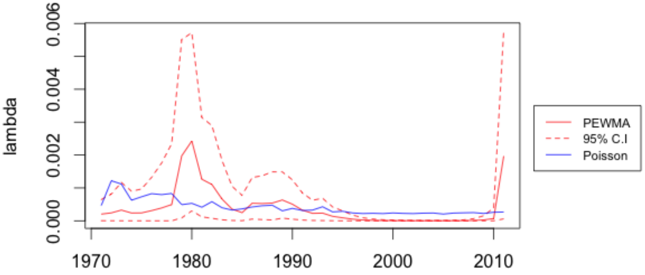
The Gamma parameters are updated following a simple formula:

$$\lambda_t|Y_t \sim \Gamma(a_t, b_t)$$

Where:

- $a_t = a_{t|t-1} + y_t$
- $b_t = b_{t|t-1} + E_t$

Poisson vs PEWMA



PEWMA Results

Model	$\hat{\lambda}_{2011}$	$\Delta_{2011-2010}\%$
Poisson	0.00026	2.76%
PEWMA	0.00196	270%
Δ	-86.4%	

- Poisson estimate does not react substantially to Fukushima Dai-ichi, whereas in PEWMA model it is the most important event to predict the arrival rate
- The Poisson arrival rate is 86 % smaller than PEWMA rate for 2011
- Fukushima Dai-ich event represented an important increase in PEWMA estimation (270%)

Probability of an accident and Fukushima Dai-ichi effect

Let's compare the results that from the models that we have discussed so far. When we compute at the end of 2011, the worldwide (433 nuclear reactors) probability of at least one nuclear accident for next year and the increase in the rate due to Fukushima Dai-ichi we find the following results:

Model	Frequency p	Worldwide $1 - P(k = 0)$	Fukushima Effect
Binomial	0.00076	0.28	0.37
Bayesian Binomial Beta	0.00032	0.12	0.25
Model	Arrival rate λ	Worldwide $1 - P(k = 0)$	Fukushima Effect
Poisson regression	0.00026	0.0003	0.027
PEWMA	0.00196	0.0020	2.70

Conclusions

- The PEWMA model is a first attempt:
    - i To reconcile the observed frequency and the estimated CDF provided by nuclear operators and safety authorities
    - ii To combine observations and other data related to safety. It is important to use other sources of information on nuclear risk to predict the probability of a major nuclear accident
    - ii To deal with the time series and count nature of our data
- According to this model the Fukushima Dai-ichi effect on the increase in predictive probability is large
    - This is not a contradiction with qualitative assessments. For instance in many countries other than Japan, regulatory capture and seismic underestimation prevail.
- There still are some limitations
    - i The low number of observations. We should use a broader definition of accident to have more observations (i.e INES 2+)
    - ii Is important to incorporate the heterogeneity across nuclear fleet (i.e localization, technology, age)

Brandt, P. and Williams, J (1998)
Modeling time series count data: A state-space approach to event counts
Unpublished paper.

Brandt, P. and Williams, J (2000)
Dynamic modeling for persistent event count time series
Unpublished paper

Cochran, T. (2011)
Fukushima nuclear disaster and its implications for U.S. nuclear power reactors
Technical report

Electric Power Research Institute (2008)
Safety and operational benets of risk-informed initiatives
Technical report

Harvey, A. and Fernandes, C. (1989)
Time series models for count or qualitative observations
Journal of Business and Economic Statistics :7, 407–417.

Harvey, A. and Shepard, N. (1993)
Structural time series models
Handbook of Statistics 11, 261–301.

U.S. Nuclear Regulatory Commission (1990)
Severe accident risks: An assessment for ve U.S. nuclear power plants.
Nureg-1150, Technical report

U.S. Nuclear Regulatory Commission (1997)
Individual plant examination program: Perspectives on reactor safety and plant performance.
Nureg-1560, Technical report

Sovacool, B. (2008)
The costs of failure:a preliminary assessment of major energy accidents 1907-2007
Energy Policy 36, 1802–1820

Corchan Nuclear Accidents

Year	Location	Unit	Reactor type
1959	California, USA	Sodium reactor experiment	Sodium-cooled power reactor
1966	Michigan, USA	Enrico Fermi Unit 1	Liquid metal fast breeder reactor
1967	Dumfreshire, Scotland	Chapelcross Unit 2	Gas-cooled, graphite moderated
1969	Loir-et-Chaire, France	Saint-Laureant A-1	Gas-cooled, graphite moderated
1979	Pennsylvania, USA	Three Mile Island	Pressurized Water Reactor (PWR)
1980	Loir-et-Chaire, France	Saint-Laureant A-1	Gas-cooled, graphite moderated
1986	Pripyat, Ukraine	Chernobyl Unit 4	RBKM-1000
1989	Lubmin, Germany	Greifswald Unit 5	Pressurized Water Reactor (PWR)
2011	Fukushima, Japan	Fukusima Daiichi Unit 1	Boiling Water Reactor (BWR)
2011	Fukushima, Japan	Fukusima Daiichi Unit 2	Boiling Water Reactor (BWR)
2011	Fukushima, Japan	Fukusima Daiichi Unit 3	Boiling Water Reactor (BWR)
			Cochran (2011)

Figure: Major nuclear accidents 1955-2011

Go back

Poisson Exponentially Weighted Moving Average (PEWMA)

1. Measurement equation: Is the stochastic component $f(y_t|\lambda_t)$, we keep our assumption that y_t is distributed Poisson with arrival rate λ_t

$$\lambda_t = \lambda^*_{t-1} \frac{\exp(X'_t\beta)}{E_t}$$

The rate has two components:

- An unobserved component λ^*_{t-1}
- A log-link like in Poisson regression

PEWMA Model equations

2. Transition equation: Shows how the mean changes over time.

$$\lambda_t = \lambda_{t-1} \exp(r_t)\eta_t$$

- r_t is the rate of growth
- η_t is a random shock that is distributed $B(a_{t-1}\omega, (1 - a_{t-1})\omega)$
- ω is weighting parameter. When $\omega \rightarrow 1$ observations are independent, $\omega \rightarrow 0$ the series is persistent

3. Prior distribution: Describes the initial state.

$$\lambda^*_{t-1} \sim \Gamma(a_{t-1}, b_{t-1})$$

- We are going to use the conjugate for the Poisson that a Gamma distribution

Kalman filter procedure

We are interested in finding:

$$f(y_t|Y_{t-1}) = \int_0^\infty \underbrace{f(y_t|\lambda_t)}_{Measurement} \underbrace{\pi(\lambda_t|Y_{t-1})}_{Unknown} d\lambda_t$$

So to find $\pi(\lambda_t|Y_{t-1})$ we use a Kalman filter. Following these steps:

1. Combine the prior distribution of λ_{t-1} with the transition equation to find the distribution of $\lambda_t|Y_{t-1}$
2. Using the properties of the gamma distribution we find the parameters $a_{t|t-1}, b_{t|t-1}$
3. We use the Bayes' updating formula to compute the distribution of $\lambda_t|Y_t$ whenever the information set is available (i.e $\forall t < T$)
4. This updated distribution becomes the prior in the next period and we repeat the previous steps

Posterior distribution for λ_t

When we combine the transition function with the prior is possible to show that:

$$\lambda_t | Y_{t-1} \sim \Gamma(a_{t|t-1}, b_{t|t-1})$$

Where:

- $a_{t|t-1} = \omega a_{t-1}$
- $b_{t|t-1} = \omega b_{t-1} \frac{\exp(-X_t' \beta - r_t)}{E_t}$

The posterior distribution is also Gamma and the parameters are updated following a simple formula:

$$\lambda_t | Y_t \sim \Gamma(a_t, b_t)$$

Where:

- $a_t = a_{t|t-1} + y_t$
- $b_t = b_{t|t-1} + E_t$

Log-likelihood function

Now we can compute $f(y_t|Y_{t-1})$ that is given by a negative binomial density function

$$\begin{aligned} f(y_t|Y_{t-1}) &= \int_0^{\infty} f(y_t|\lambda_t)\pi(\lambda_t|Y_{t-1})d\lambda_t \\ &= \frac{\Gamma(\omega a_{t-1}+y_t)}{y_t!\Gamma(\omega a_{t-1})}\{\omega b_{t-1}\frac{\exp(-X_t'\beta-r_t)}{E_t}\}^{\omega a_{t-1}} \\ &\quad \times\{E_t+\omega b_{t-1}\frac{\exp(-X_t'\beta-r_t)}{E_t}\}^{-(\omega a_{t-1}+y_t)} \end{aligned}$$

So the predictive joint distribution is

$$f(y_0, ..., Y_T) = \prod_{t=0}^{T} f(y_t|Y_{t-1})$$

The log-likelihood function is based on this joint density

$$\mathcal{L} = \log(f(y_0, ..., Y_T))$$

Go back

Results

Table: PEWMA results

	Coefficient	Std.Error	Z-Score
ω	0.8477619	0.01338405	63.341200
constant	-7.7252291	0.50184279	-15.393723
UUF	0.1588987	0.04180016	3.801391

- $\hat{\omega}$ can be consider as a smoothing or discounting parameter.
- $\hat{\omega}$ different from 1 means that we reject the independence hypothesis. Small values indicate more dynamics so nearest observations are more important than those in the past.
- $\hat{\beta}$ estimates have the same sign that in the Poisson model, which confirms a decreasing time trend in nuclear accidents and reductions in UUF are linked with smaller nuclear risk.

Go back