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CIRED – Centre International de Recherche sur l'Environnement et le Développement

Working Paper N°2016-05-23

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August 31, 2016

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1 Introduction

The IMACLIM modeling approach has been developed at CIRED since the early 90s. At the core of the blueprint has been the objective to build hybrid modeling architectures to articulate energy system and economy-wide representations to explore energy-climate-economy futures (Hourcade, 1993; Hourcade et al., 2006; Ghersi and Hourcade, 2006). Currently, the IMACLIM approach comes in several modeling platforms including a global multi-region recursive-dynamic version - IMACLIM-R WORLD (Sassi et al., 2010; Bibas et al., 2015) and national versions for France either recursive-dynamic - IMACLIM-R FRANCE (Bibas, 2013) - or in comparative statics - IMACLIM-S FRANCE (Combet, 2013). Recently, several national versions have been developed mainly for emerging economies like Brazil and South Africa. The development of the IMACLIM platform for Brazil - called IMACLIM-BR is the result of an ongoing collaboration with the COPPE of the University of Rio de Janeiro.

IMACLIM-BR is a hybrid CGE modeling platform of the Brazilian economy specifically designed to build consistent projections of the energy-GHG emissions-economy system in Brazil over the medium to long term. It makes it especially possible to assess the macroeconomic implications of price or quantity-based carbon and energy policy. IMACLIM-BR departs from more standard neoclassical CGE models in several features.

First of all, like standard CGE models, IMACLIM-BR rests upon the representation of walrasian markets of goods and services with global income balance. In addition, like most hybrid CGE models, IMACLIM-BR is based on an energy-economy hybrid accounting framework where economic flows *and* physical flows (with a special focus on energy balances) are balanced for the issue of energy-economy hybrid accounting frameworks). However it first departs from the neoclassical approach in that its description of the consumers' and producers' trade-offs, and the underlying technical systems, are specifically designed to facilitate calibration on bottom-up expertise in the energy field, with a view to guaranteeing technical realism to the simulations of even large departures from base year point.

Second, IMACLIM-BR represents "second best" economic systems - distant from the canonical competitive economy - and computes future accounting balances and walrasian markets of goods and services characterized by possible underemployment of production factors (labor) and imperfect markets (goods and factors). To do so, the model relies on a specific representation of capital and on other structural assumptions. In this feature it can be related to the tradition of structuralist CGE models (Taylor, 1990).

Third, IMACLIM-BR computations rely on the method of comparative statics (Samuel-

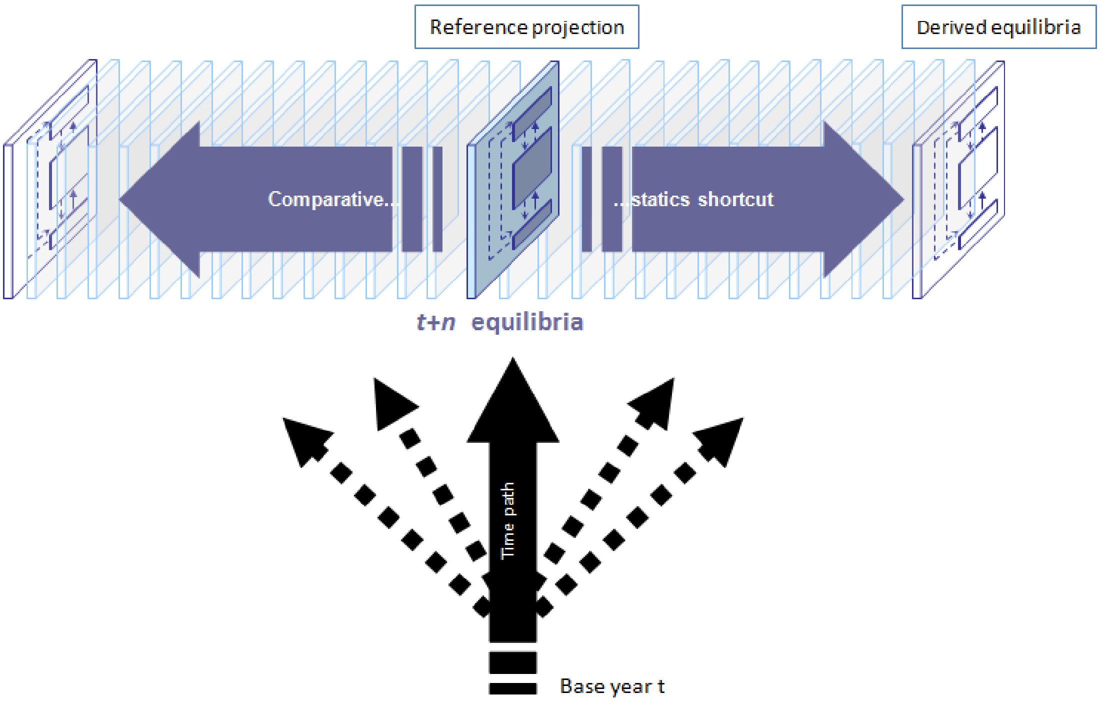

Figure 1: Single-step projections and comparative statics

son, 1983; Kemfert, 2003): the model generates medium-run energy-economy projections in a single time step and compares, at the time horizon studied, these different projections as different counterfactuals driven by alternative sets of parameters and policy packages. The insights provided are valid under the assumption that the policy-induced transition from base year to a given policy-constrained future, is completed, after a series of technical and economic adjustments whose duration and scope are embedded in the behavioral functions retained for the time horizon under consideration. The transition process itself is however not described, but implicitly supposed to be smooth enough to prevent e.g. multiple equilibria, hysteresis effects, etc.

This working paper provides a full description of the IMACLIM-BR model. Section 2 characterizes the model in a compact format with its key equations in order to highlight its main specifics. Section 3 provides a comprehensive description and formulary of the central model version. Section 4 details the specifics of two expanded versions.

2 Characterization of the model in a compact format

We provide in this section a synoptic characterization of the model.

2.1 Key equations

The comparative statics framework boils down to a set of simultaneous equations at the time horizon studied:

$$\begin{cases} f_1\left(x_1, \cdots, x_n, \overline{\phi_1}, \cdots, \overline{\phi_m}\right) = 0 \\ f_2\left(x_1, \cdots, x_n, \overline{\phi_1}, \cdots, \overline{\phi_m}\right) = 0 \\ \cdots \\ f_n\left(x_1, \cdots, x_n, \overline{\phi_1}, \cdots, \overline{\phi_m}\right) = 0 \end{cases}$$

$(x_i)_{i\in\{1,\cdots,n\}}$ are the variable of the model.
$\left(\overline{\phi_i}\right)_{i\in\{1,\cdots,m\}}$ is a set of exogenous parameters.
$(f_i)_{i\in\{1,\cdots,n\}}$ is a set of exogenous functions, linear or non-linear.

The f_i constraints are of two quite different natures: (i) one subset of equations describes the accounting constraints that are necessarily verified to ensure that the accounting system is properly balanced and (ii) the other subset translates the technical and economic choices.

A compact version of model is made of the following blocks:

- Domestic price formation:

$$\textit{Price of production} \qquad P_Y = P \cdot \alpha + w \cdot l + p_K \cdot k + p_{LD} \cdot ld + P_Y \cdot diag\,(\Pi) \tag{1}$$

$$\textit{Average price} \qquad P = \left[\frac{p_{Yj} \cdot Y_j + \overline{p_{Mj}} \cdot M_j}{Y_j + M_j}\right]_j \tag{2}$$

- Income generation and usage:

$$\textit{Closure rule} \qquad R_{CONS} + R_{INV} = \overline{sh} \cdot \left[\sum_j \left(w \cdot l_j + p_K \cdot k_j + p_{LD} \cdot ld_j + p_{Yi} \cdot \pi_j\right) \cdot Y_i\right] \tag{3}$$

$$\textit{Investment} \qquad R_{INV} = p_{COMP} \cdot I_{COMP} \tag{4}$$

- Demographic driver:

$$\textit{Active population} \qquad NS = NS \cdot \left(1 + \overline{\delta_{NS}}\right) \tag{5}$$

- Productivity drivers and production trades-off:

$$\textit{Input intensity of production} \qquad \alpha = \left[f_{ij}\left(P, w, p_K, p_{LD}, \overline{\phi_{ij}}\right)\right]_{ij} \tag{6}$$

$$\textit{Labor intensity of production} \qquad l = \left[f_{Lj}\left(P, w, p_K, p_{LD}, \overline{\phi_{Lj}}\right)\right]_{j} \tag{7}$$

$$\textit{Capital intensity of production} \qquad k = \left[f_{Kj}\left(P, w, p_K, p_{LD}, \overline{\phi_{Kj}}\right)\right]_{j} \tag{8}$$

$$\textit{Land intensity of production} \qquad ld = \left[f_{LDj}\left(P, w, p_K, p_{LD}, \overline{\phi_{LDj}}\right)\right]_{j} \tag{9}$$

- Final consumption trades-off:

$$C = \left[f_{Cj}\left(R_{CONS}, P\right)\right]_j \tag{10}$$

$$\tag{11}$$

- Breakdown of value-added:

$$\textit{Wage curve} \qquad w = f_w\left(1 - \frac{L}{NS}\right) \tag{12}$$

$$p_K = p_{COMP} \tag{13}$$

$$p_{LD} = \overline{p_{LD}} \qquad \text{or} \qquad p_{LD} = g_{LD}\left(LD\right) \tag{14}$$

$$\textit{Mark-up pricing} \qquad \Pi = \overline{\Pi} \tag{15}$$

- Trade:

$$X = \left[f_{Xj}\left(p_{Yj}, p_{Mj}, \overline{X_j}\right)\right]_j \tag{16}$$

$$M = \left[f_{Mj}\left(p_{Yj}, p_{Mj}, Y_i\right)\right]_j \tag{17}$$

- Markets - accounting balances in volumes:

$$\textit{Market of goods} \qquad Y = Y \cdot \alpha^t + C + I + X - M \tag{18}$$

$$\textit{Labor market} \qquad L = \sum_j l_j \cdot Y_j \tag{19}$$

$$\textit{Implicit capital balance} \qquad \sum_j k_j \cdot Y_j = \overline{\beta} \cdot I_{COMP} \tag{20}$$

$$\textit{Land market} \qquad LD = \sum_j ld_j \cdot Y_j \tag{21}$$

2.2 A set of distinctive features

2.2.1 A SAM-based CGE model

First of all IMACLIM-BR is a CGE model with multiple economic sectors based on a standard Social Accounting Matrix (SAM) and a square Input-Output matrix (α represents the matrix of technical coefficients). It follows the Arrow-Debreu formulation of volumes (Y, C, I, X, M, L, LD (land), etc) and prices ($p_Y, p, p_M, w, p_K, p_{LD}$) for goods and production factors, solves walrasian markets of goods and insures the usual accounting identities of the SAM in volumes and money flows for the different markets (goods and factors) and institutional sectors budgets (one single representative domestic agent in this compact version and the rest of the world). It also represents an opened economy with trade effects and the closure rule (in this compact version only) is based on an exogenous trade balance ratio in total domestic income ($\overline{sh}$). Eventually production and consumption trades-off are modeled by means of aggregated functions (f). The model includes three factors of production (labor(L), capital (K) and land (LD)).

2.2.2 An exogenous growth engine with implicit capital accumulation

The first specific of the model pertains to its growth engine. Usually, CGE models are either "static" or "recursive dynamic". In the first case, the model simulates counterfactuals at a given date with fixed labor and capital endowments. There is no growth model and final demand of investment goods do not increase capital endowment. In the case of recursive-dynamic CGE models, the growth engine is usually based on an exogenous growth model with *explicit* recursive capital - investments dynamics, a demographic driver and exogenous technical change (Harrod's neutral technical change in the example below based on labor productivity gains):

$$K_t = (1 - \delta) \cdot K_{t-1} + I_t \tag{22}$$

$$L_t = L_0 \cdot e^{\delta \cdot t} \tag{23}$$

$$Y_j = f\left(K, e^{\phi_L \cdot t} \cdot L, E, M\right) \tag{24}$$

Our model aims at exploring future states of the economy in the medium and long run, so it needs to model economic growth. To do so, it includes a growth engine based on usual demographic drivers and technical change features but with an *implicit* representation of capital accumulation, else consistent with the comparative statics framework and the generation of future states of the economy in one single step.

Therefore, although the model's computations rely on comparative statics, it is not a static CGE (in the sense of fixed factors endowment) and it does model economic growth in the medium to long run.

In practice, as in a standard growth model, it includes both an exogenous demographic driver and factor augmenting productivity coefficients ($\overline{\phi}$) consistent with the time horizon studied. However, it substitutes for the equation of *explicit* capital stock dynamics, a simpler link of proportionality between a proxy of total capital stock (total fixed capital consumption - $\sum_j k_j \cdot Y_j$) and real investment flows (fixed capital formation - I_{COMP}) at the year of projection: $\sum_j k_j \cdot Y_j = \overline{\beta} \cdot I_{COMP}$. This feature in fact surmises a smooth economic growth between base year and the time of horizon for a given projection. It should be noted that this does not mean that the economy is on a stabilized growth path in Solow's sense as a perpetual steady state. On the contrary, the model aims at representing unbalanced trajectories with sub-optimal employment of factors. The model just circumvents the representation of complex transitional sequences and averages a growth rate between base-year and the time of projection for more simplicity.

2.2.3 A reinterpretation of the production function

In most CGE models, production trade-offs are modeled by means of production functions. Historically production functions have been used to represent the trade-off between production factors according to their relative prices in the long run along steady growth pathways. However the same production functions are used to model production trade-off in intra-temporal equilibrium (short to medium run) of recursive dynamic CGE models. Following (Ghersi and Hourcade, 2006), IMACLIM-BR embodies a renewed interpretation of the production function as the envelop of technical choices respective to a given time horizon. The envelop of technical choices per sector is embodied by the set of functions f_{ij}, f_{Lj}, f_{Kj}, f_{LDj} (see the equations in section 2.1). As in fig. 2, technical choices at $t+n$ are the results of given relative prices regimes between t and $t+n$ and can include implicit price-induced technical change[1].

The resulting envelop of technical choices or "innovation possibility curve" (Ahmad, 1966) is eventually the envelop of all the possible isoquants linked to different relative prices regimes. This interpretation of the production frontier at a given time horizon provides new options for bottom-up - top-down hybridization as detailed in **??**. In this view, the envelop of production can be built as a reduced form of bottom-up

[1] A durable regime of high relative price of factor 1 will trigger biased technical change towards a relative decrease of factor 1 intensity in production.

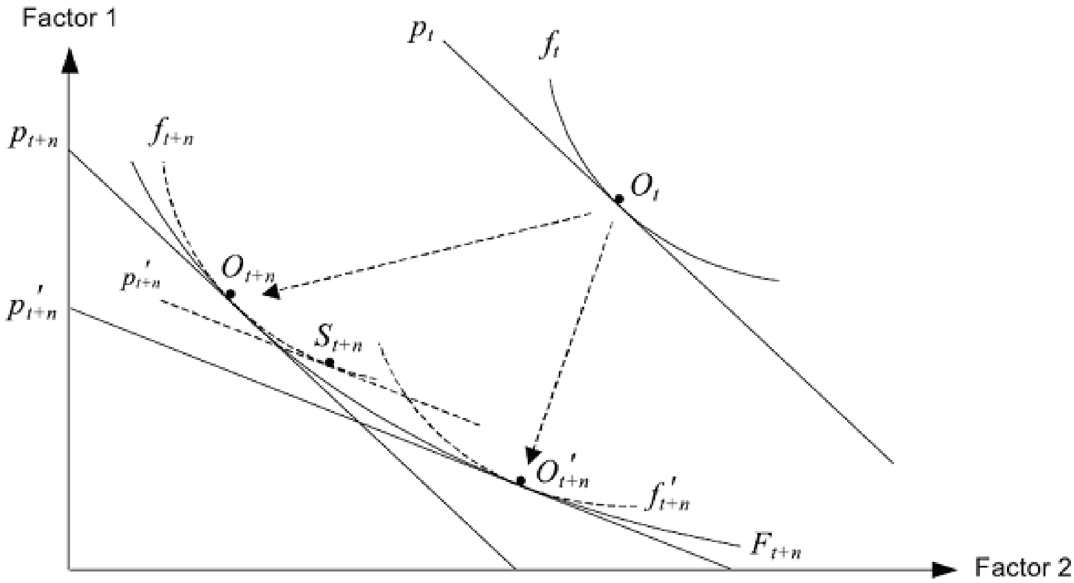

Figure 2: Production frontier and innovation possibility curve

Source: (Ghersi and Hourcade, 2006)

projections and thus mimic possible bottom-up singularities beyond constant elasticity of substitution. One prerequisite to implement this method is to keep physical accounting in the CGE model in order to embark the technical information provided by the bottom-up model (about energy efficiency for instance). In practice, few complete reduced forms have been mainstreamed in Imaclim-S models. In Ghersi et al. (2011) for instance, the Imaclim-S France model embarks bottom-up information about technical asymptotes in standard CES functions. However, this is a simple and convenient mean to inform some technical boundaries and imply non-constant elasticities of substitution as the system gets closer to its technical limits. For preliminary exercises not reported in this thesis, we have implemented reduced forms of MAC curves for industry sectors in a specific version of Imaclim-BR (Wills, 2013). The modeling choices for the versions of Imaclim-BR used in this thesis are discussed in section 3.5.2.

2.2.4 Specific capital and labor markets and imperfect markets of goods

An other specific of Imaclim-BR is the representation of capital. The capital "consumption" for production $K = k.Y$ is not modeled as the standard capital stock and the model does actually not track capital stock. In fact, the capital factor of production in real terms corresponds to the fixed capital consumption or the depreciation of capital as a proxy of capital stock. In addition, no explicit capital market is modeled and production sectors do not trade-off capital with other production factors according to a standard return on capital. Instead, trade-offs about the capital content of production are based on the price of the composite good (a weighted average of the different investment goods in the expanded model) as the price of the "machine" capital ($p_K = p_{COMP}$). The return on capital (as the economic productivity of capital) beyond depreciation is aggregated into a sector specific mark-up (Π coefficients). The underlying idea is to depart from the standard assumption about a capital market where the return on capital equals its marginal productivity, which is often a heroic assumption taken from microeconomic theory. In addition the marginal assumption is generally coupled with the zero-profit hypothesis, so that pure profit does not exist (with constant return to scale). Instead, we choose to model capital choices based on the technical content of capital (the "machine"). The reminder of the value-added beyond pure capital consumption, which corresponds to profits in the larger sense, is modeled with a mark-up. This mark-up also reflects different trend levels of goods markets imperfections (oligopolistic or monopolistic trends for instance). As a whole no explicit capital market is modeled and capital "consumption" determines investment needs at the year of projection through the β coefficient or vector.

In addition, the model includes several features about labor markets that depart from standard assumptions. First of all, the model measures the labor factor in full time equivalent jobs and does not derive it from benchmark labor cost shares. This has important implications for the relative levels of apparent labor productivity across sectors. Finally, Imaclim-BR includes the representation of involuntary unemployment and trend rigidities in the medium run on labor markets by means of a global wage curve (Blanchflower and Oswald, 1995). This wage curve is an empirical relationship between the average wage (whether nominal or real wage depending on the assumption) and the global unemployment rate of the economy.

2.2.5 Dual accounting and hybrid I-O framework

The consistency of projections is managed through maintaining a double account of flows both in physical and value units when generating the projections. As mentioned, it also enables to embark bottom-up information in an explicit manner. For this to be valid, the model should only include modeling features that maintain balance of physical volumes. For instance traded energy goods are supposed to be perfect substitutes and do not follow standard Armington's assumption as opposed to non-energy goods. Keeping up double accounting eventually supposes an important data work to build the initial state of the system. **??** shows the impact of data treatment on modeling results.

2.3 Positioning of Imaclim-BR in CGE modeling

In the literature of climate policy assessment and energy-economy models, CGE modeling usually refers to a narrowed type of modeling approaches, although majority, inspired by the micro-economic theory of Arrow-Debreu general equilibrium. In this dominant approach inherited from recent decades, economic behaviors are supposed to be micro-founded and to reflect rigorous micro-economic theory based on surplus maximization of economic agents and market (goods and factors) clearing. In practice economic behaviors are most of the time informed by constant return-to-scale CES production functions and markets are assumed perfect with zero profit conditions (Wing, 2004). The underlying goal is to embody the competitive equilibrium of micro-economic theory in empirical macroeconomic models. Sub-optimum is possible in this framework though, through tax distortion, market failures or market power but economic behaviors remain based on rigorous micro-economic theory.

However historically, CGE models are first and foremost macro-balancing models[2] and the accounting balance of the SAM is the primary characteristic of a CGE model. In particular, accounting and income balance do not necessarily imply market clearing or full employment of factors, which constitute very specific cases. Thissen (1998) provides a larger definition of CGE modeling in this direction: "*A CGE model may be defined as the fundamental macroeconomic general equilibrium links among incomes of various groups, the pattern of demand, the balance of payments and a multi-sector production structure*". In a word, CGE models are primarily macro-balancing models before possible empirical translation of general equilibrium micro-economic theory.

Imaclim models, and Imaclim-BR especially, fit into this broader vision of CGE modeling and emphasize the idea of macroeconomic *interdependence* between demand patterns, inter-industry structure and income flows from decentralized representative economic agents. In this view the mechanisms of interdependence are fundamentally constrained by supply-use equilibrium and the accounting balance of the SAM, which is the base of the equilibrium concept. In addition, Imaclim models build on the price-quantity formulation of flows and stocks of goods and production factors to generate consistent energy-economy projections and control underlying technical systems.

[2]Leif Johansen is often credited with being the first CGE modeler and has proposed a model that combines macro-balancing equations with a Leontief production structure (Johansen, 1960). This stream of modeling significantly developed during the 70's (Taylor and Black, 1974; Adelman and Robinson, 1978). The inclusion of a link to Arrow-Debreu micro-economic theory happened only later in the 80's through the connection with Applied General Equilibrium (AGE) modeling based on Scarf's simplex method (Shoven and Whalley, 1972).

Beyond accounting balance and dual accounting of energy flows, Imaclim-BR is based on the computation of walrasian markets of goods (with simultaneous price and quantity adjustment) in the medium to long run combined with and implicit growth engine. However it abandons the systematic reference to rigorous micro-economic theory to the benefit of structural assumptions and empirical realism. Economic behaviors are represented by structural assumptions embedded in the functional forms retained. These functions can be reduced forms of bottom-up models when it is relevant. Structural assumptions also pertain to the structure of markets like for instance the abandonment of the representation of a capital market balanced with a rate of return on capital. These structural assumptions make it finally possible to represent specific second-best trends of the economy such as imperfect markets of goods through mark-up pricing (relaxing the zero-profit condition) and imperfect labor markets with involuntary unemployment.

The modeling approach retained is finally consistent with the forward-looking analysis contemplated. The primary modeling objective is to generate consistent energy-economy futures pending interdependence constraints (final demand and trade patterns, production structure, income distribution, etc.), second-best mechanisms (imperfect markets, labor market rigidities, etc.) and technical constraints (technical change, substitution possibilities, etc.). The approach favors variant analysis about structural assumptions informed by stakeholders dialog (engineers and economists for instance) and swaps part of the internal rigor for higher expected empirical relevance.

3 The complete modeling features of the reference version

This section details the complete features and equations of the reference version of Imaclim-BR , which distinguishes 6 economic sectors - referred to as Imaclim-BR 6 or simply Imaclim-BR . Next section develops the specifics of the 6-sectors version detailing 6 households groups - Imaclim-BR 6-6 - and the 12-sectors version with an expanded demand system and focus on oil, liquid fuels and transport sectors -Imaclim-BR 12-ext. The three model versions are used to provide the analysis of the last two chapters.

3.1 Accounting framework: SAM (Social Accounting Matrix)

The Imaclim-BR versions used in this thesis, distinguish 6 or 12 productive sectors (table 1). The 6-sectors versions consider three energy sectors (bioenergy, fossil fuels and electricity) and three end-use sectors (agroindustry, general industry and services

Sectors		
6-sectors version	**12-sectors version**	**Production factors**
Energy		Labor
Bioenergy (BIO)	Bioenergy (BIO)	Capital
Fossil fuels (FF)	Coal (COAL)	Land
	Crude Oil (OIL)	
	Natural Gas (GAS)	
	Refined Oil (ROIL)	
Electricity (ELEC)	Electricity (ELEC)	
Non-Energy		
Agroindustry (AGRI)	Livestock (CATT)	
	Agro-industry (AGRI)	
General industry (INDUS)	Industry (INDUS)	
Services (COMP)	Load Transportation (LOAD)	
	Pass. Transportation (PASS)	
	Services (COMP)	

Table 1: Sectors and production factors

- including transportation services). The 12-sectors version distinguish six energy sectors (bioenergy, coal, crude oil, natural gas, refined oil and electricity) and six end-use sectors (load transportation, passenger transportation, livestock, agro-industry, general industry and services). All versions include three primary production factors: labor, capital and land.

The interactions between productive sectors and factors are included in the larger accounting framework of IMACLIM-BR summarized by its SAM (see table 2).

	Sectors	**Labor**	**Capital, Profits, Margins**	**Land**	**HH**	**Firms**	**GOV**	**ROW**	**INV**	**FINCAP**	**Others**
Sectors	$p_Y \cdot IC_{dom}$				$p_Y \cdot C_{dom}$		$p_Y \cdot G_{dom}$	$p_Y \cdot X_{dom}$	$p_Y \cdot I_{dom}$		
Labor	$w \cdot l \cdot Y$										
Capital	$p_K \cdot k \cdot Y$										
Land	$p_{LD} \cdot ld \cdot Y$										
Profits	$\pi \cdot p_Y \cdot Y$										
Margins	$BM + TM + SM$				$BM + TM + SM$		$BM + TM + SM$	$BM + TM + SM$	$BM + TM + SM$		
HH		$w \cdot l \cdot Y$	$\overline{\omega_{Kh}} \cdot GOS$	$\overline{\omega_{LDh}} \cdot LAND$		$\rho_f \cdot N$	$\rho_g \cdot N$				OT_h
Firms	TL_f		$\overline{\omega_{Kf}} \cdot GOS$	$\overline{\omega_{LDf}} \cdot LAND$	T_{hf}						OT_f
GOV	$T_{CONS} + TL_g + T_Y$		$\overline{\omega_{Kg}} \cdot GOS$	$\overline{\omega_{LDg}} \cdot LAND$	$T_{CONS} + TI_h + T_{hg}$	TI_f	T_{CONS}	T_{CONS}	T_{CONS}		OT_f
ROW	$p_M \cdot IC_{imp}$				$p_M \cdot C_{imp}$		$p_M \cdot G_{imp}$	$p_M \cdot X_{imp}$	$p_M \cdot I_{imp}$		OT_m
INV					$GFCF_h$	$GFCF_f$	$GFCF_g$				
FINCAP					$FCAP_h$	$FCAP_f$	$FCAP_g$	$FCAP_m$			
Other transf.											

Table 2: Imaclim-BR Social Accounting Matrix (SAM)

Each sector produces one single good so that commodities and activities match and the Input-Output table is square. Productive sectors generate income through productive factors. The national income is further distributed to representative institutional agents in the same pattern as in national accounts. The model distinguishes 4 institutional agents or sectors: *households* (HH), *corporate firms* (FIRMS), *public administrations* or *government* (GOV) and the *rest of the world* (ROW). On the whole IMACLIM-BR keeps the orthogonal logic between productive sectors and institutional agents in order to keep the detail of primary income distribution. This makes it possible to identify the shares of capital, land income and generated profits that primary accrue specifically to autonomous producers (Households: family farms, individual entrepreneurs, landlords (agriculture and housing), etc), corporate firms or public administrations (public companies). Therefore households, firms and government have separated accounts in our model and may have different structural behaviors. CGE models usually shortcut this aspect by assuming that households eventually own the total endowment of production factors. Only households are endowed with labor in the model.

Furthermore, through secondary income distribution, institutional agents break down their income between goods consumption, investment, tax payments and transfers. The model considers a detailed system of taxes and transfers essentially between the triangle of domestic agents. We will detail this system later in the model description. Owing to the split of accounts of institutional agents, IMACLIM-BR also considers the breakdown of total *gross fixed capital formation* (GFCF) between agents. It further identifies for each agent the share of income that is not directly invested in GFCF, which is called financial capacity (FCAP). The rest of the world classically interacts with domestic agents through trade of goods and capital balance.

The rest of the section details the equations of the model through different blocks: (i) *price system and income generation*, (ii) *institutional sectors accounts*, (iii) *production and households consumption trade-off* and (iv) *market balances*. The equations of the model are of two quite different natures: one subset of equations describes accounting constraints that are necessarily verified to ensure that the accounting system is properly balanced; the other subset translates various behavioral constraints, written either in a simple linear manner (e.g. households consume a fixed proportion of their income) or in a more complex non-linear way (e.g. the trade-offs of production and consumption). It is these behavioral constraints that ultimately reflect, in the flexible architecture of IMACLIM-BR a certain economic "worldview".

The equations distinguish three kinds of components: (i) the variables computed by the model, which represent the endogenous elements of the projected energy-economy picture at the time horizon studied, (ii) the parameters that are calibrated on the base year SAM and (iii) the other non calibrated parameters coming from external sources. In the equations, the calibrated parameters are identified with an over-line and base year variables are indexed with 0. Although most equations are written in an generalised n-goods format, when necessary good-specific variables are indexed by the subscripts detailed in table 1. Variables names are also consistent with the SAM.

3.2 Price system - Income generation

As already introduced with the compact form of IMACLIM-BR , prices and income generation are first channels to impose structural constraints in the model as a first source of departure from a neoclassical CGE model. It includes (i) the representation of non-zero profits through mark-up pricing, (ii) the inclusion of specific margins for energy goods, (iii) sector specific wages and (iv) a specific price for capital consumption.

3.2.1 Prices

First of all, p_{Yj}, the producer's price of good-sector j is following the cost structure of the production of good j plus a pure profit component. p_{Yj} is thus built as the sum of intermediate consumptions, labour costs, capital costs, land costs (for land-use sectors only) a tax on production, and a rate of profit.

$$p_{Yj} = \sum_{i=1}^{n} p_{ICij} \cdot \alpha_{ij} + p_{Lj} \cdot l_j + p_k \cdot k_j + p_{LD_j} \cdot ld_j + \overline{\pi_j} \cdot p_{Yj} + \overline{\tau_{Yj}} \cdot p_{Yj} \tag{25}$$

Technical coefficients α, l, k and ld are expressed in real terms. Technical coefficients for intermediary consumption of energy are specifically expressed in ktoe per unit of output.

The rate of profit $\overline{\pi}$, which corresponds in practice to the net operating surplus, is constant and calibrated at base year for all sectors in the reference version.

This mark-up pricing is used to translate both the specific structural conditions of the different market of goods and all costs that are not pure capital consumption costs. We detail the meaning of this capital consumption in section 3.5. The specific structural conditions embody the departure from the perfect competitive case of a first best setting.

p_{Mi} the price of imported good j is good-specific and the international composite good is the *numéraire* of the model; its price is assumed constant and equal to unity.

$$p_{M_{COMP}} = p_{M_{COMP_0}} = 1 \tag{26}$$

The prices of others goods evolve according to an exogenous rate $\delta_{p_{Mi}}$:

$$p_{Mi} = \left(1 + \delta_{p_{Mi}}\right)^t \cdot p_{Mi_0} \tag{27}$$

$\delta_{p_{Mi}}$ parameters is used to simulate alternative world energy prices scenarios.

CGE models usually adopt the assumption of goods differentiation between domestic and imported goods and the implementation of an Armington specification (Armington, 1969). However the Armington specification has the disadvantage of creating "hybrid" good varieties, whose volume unit is independent from that of the foreign and national varieties they hybridize. This prevents to maintain an explicit accounting of the physical energy flows and thus an energy balance. Consequently, in order to keep the account of physical volumes (which is a fundamental of our modeling approach), imported and domestic energy goods are assumed to be homogeneous:

$$p_i = \frac{p_{Yi} \cdot Y_i + p_{Mi} \cdot M_i}{Y_i + M_i} \tag{28}$$

Nevertheless, imported and domestic energy goods can coexist in the domestic market even with different prices (see subsection 3.6). For the sake of simplicity, non-energy goods are treated similarly.

p_{ICij} the purchaser's price of good i consumed for the production of good j, is equal to the resource price of good i plus trade and transport margins, specific margins and a rate of aggregate *ad valorem* tax on consumption (sales tax):

$$p_{ICij} = p_i \cdot \left(1 + \tau_{BMi} + \tau_{TMi} + \overline{\tau_{MSICij}}\right) \cdot (1 + \overline{\tau_{CONSi}}) \tag{29}$$

The purchaser's price of good i for households consumption (p_{Ci}), public administrations (p_{Gi}) and investment (p_{Ii}), and the export price of good i (p_{Xi}), are constructed similarly[3]:

$$p_{Ci} = p_i \cdot (1 + \tau_{BMi} + \tau_{TMi} + \overline{\tau_{MSCi}}) \cdot (1 + \overline{\tau_{CONSi}}) \tag{30}$$

$$p_{Gi} = p_i \cdot (1 + \tau_{BMi} + \tau_{TMi} + \overline{\tau_{MSGi}}) \cdot (1 + \overline{\tau_{CONSi}}) \tag{31}$$

$$p_{Ii} = p_i \cdot (1 + \tau_{BMi} + \tau_{TMi} + \overline{\tau_{MSIi}}) \cdot (1 + \overline{\tau_{CONSi}}) \tag{32}$$

$$p_{Xi} = p_i \cdot (1 + \tau_{BMi} + \tau_{TMi} + \overline{\tau_{MSXi}}) \tag{33}$$

One exception is the export price of fossil fuels (FF) which is indexed on world

[3] p_{ICij} and p_{Ci} are possibly increased by a carbon tax as detailed in subsection 3.8.

price:

$$p_{X_{FF}} = p_{M_{FF}} \cdot \frac{p_{X_{FF0}}}{p_{M_{FF0}}} \tag{34}$$

We also introduce the pre-tax price system:

$$p_{BTICij} = p_i \cdot \left(1 + \tau_{BMi} + \tau_{TMi} + \overline{\tau_{MSICij}}\right) \tag{35}$$

$$p_{BTZi} = p_i \cdot (1 + \tau_{BMi} + \tau_{TMi} + \overline{\tau_{MSZi}}) \text{ with } Z \in \{C, G, I\} \tag{36}$$

Specific margins are calibrated at base year and held constant to reflect the difference of tariffs (taxes excluded) of energy goods according to the different consuming agent/sector.

Business margins τ_{BMi} and transport margins τ_{TMi}, identical for all intermediate and final consumption of good i, are calibrated at base year and kept constant - except those on freight transport and trade activities aggregated within the *COMP* sector, which are simply adjusted, to have the two types of margins sum up to zero :

$$\begin{cases} \tau_{BM_{COMP}} = \overline{\tau_{BM_{COMP}}} \\ \tau_{TM_{COMP}} = \overline{\tau_{TM_{COMP}}} \end{cases} \tag{37}$$

$$\sum_{j=1}^{n} \tau_{BM_{COMP}} \cdot p_{COMP} \cdot \alpha_{COMPj} \cdot Y_j + \tau_{BM_{COMP}} \cdot p_{COMP} \cdot (C_{COMP} + G_{COMP} + I_{COMP} + X_{COMP}) + \sum_{i \neq COMP} \sum_{j} \overline{\tau_{BMi}} \cdot p_i \cdot \alpha_{ij} \cdot Y_j + \sum_{i \neq COMP} \overline{\tau_{BMi}} \cdot p_i \cdot (C_i + G_i + I_i + X_i) = 0 \tag{38}$$

$$\sum_{j=1}^{n} \tau_{BM_{COMP}} \cdot p_{COMP} \cdot \alpha_{COMPj} \cdot Y_j + \tau_{BM_{COMP}} \cdot p_{COMP} \cdot (C_{COMP} + G_{COMP} + I_{COMP} + X_{COMP}) + \sum_{i \neq COMP} \sum_{j} \overline{\tau_{BMi}} \cdot p_i \cdot \alpha_{ij} \cdot Y_j + \sum_{i \neq COMP} \overline{\tau_{BMi}} \cdot p_i \cdot (C_i + G_i + I_i + X_i) = 0 \tag{39}$$

Imaclim-BR accounts for labor service in full-time equivalent jobs and thus deals with sector specific wages. Labor costs are further equal to the sector specific net wage w_i plus payroll taxes that correspond to employers social contributions for private and public employees social care (pensions, health care, etc). These taxes are levied based on average sector specific rates and calibrated at base year:

$$p_{Li} = \left(1 + \overline{\tau_{Lfi}} + \overline{\tau_{Lgi}}\right) \cdot w_i \tag{40}$$

In addition, wages evolve with a common rate of change:

$$w_i = a \cdot w_{i_0} \tag{41}$$

The average wage - defined by:

$$w = \frac{\sum_{i=1}^{n} w_i \cdot l_i \cdot Y_i}{\sum_{i=1}^{n} l_i \cdot Y_i} \tag{42}$$

is subject to variations that are dictated by the supply side of labor markets which relates, by means of a wage curve, the average wage to the average rate of unemployment of the economy (see labor market balance in subsection 3.7).

The cost of capital is understood as the cost of the "machine" capital (see the description of the production trade-offs in subsection 3.5). It is obtained as the average price of investment goods:

$$p_K = \frac{\sum_{i=1}^{n} p_{Ii} \cdot I_i}{\sum_{i=1}^{n} I_i} \tag{43}$$

As mentioned earlier, it is a specific of our modeling approach where a standard CGE model compute a rate of return on capital.

3.2.2 Gross operating Surplus

Capital costs, profits and specific margins determine the gross operating surplus (*GOS*) (income from land excluded) of the economy:

$$GOS = \sum_{i=1}^{n} \left(p_K \cdot k_i \cdot Y_i + \overline{\pi_i} \cdot p_{Yi} \cdot Y_i\right) + M_s \tag{44}$$

By construction, the specific margins on the different sales M_S sum to zero in the base year equilibrium (this is a constraint of the energy-economy data hybridizing process), however they do not in the future equilibrium, their constant rates being applied to varying prices. The total specific margin generated M_S is then computed as:

$$M_s = \sum_i \left(\sum_j \overline{\tau_{MSICij}} \cdot p_i \cdot \alpha_{ij} \cdot Y_j + \overline{\tau_{MSCi}} \cdot p_i \cdot C_i + \overline{\tau_{MSGi}} \cdot p_i \cdot G_i + \overline{\tau_{MSXi}} \cdot p_i \cdot X_i \right) \tag{45}$$

The Agriculture sector (*AGRI*) generates total available land income:

$$LAND = p_{LD_{AGRI}} \cdot ld_{AGRI} \cdot Y_{AGRI} \tag{46}$$

GOS and *LAND* are further broken down between institutional sectors as described in subsection 3.3.

The consumer price index *CPI* is computed following Fisher, i.e. as the geometric mean of a Laspeyres index (variation of the cost of the present basket of goods from the present to the future set of relative prices) and a Paasche index (variation of the cost of the future basket of goods from the present to the future set of relative prices):

$$CPI = \sqrt{\frac{\left(\sum_i p_{Ci} \cdot C_{i_0}\right) \cdot \left(\sum_i p_{Ci} \cdot C_i\right)}{\left(\sum_i p_{Ci_0} \cdot C_{i_0}\right) \cdot \left(\sum_i p_{Ci_0} \cdot C_i\right)}} \tag{47}$$

3.3 Institutional sectors accounts

The equations related to institutional sectors accounts basically reflect the constraints of accounting balance embodied in the SAM. Compared to most CGE models, Imaclim-BR keeps the accounting logic of national accounts with the distinction between households, firms and public administration as different institutional sectors. Again this results in the specific break down of capital income between institutional sectors (as different legal entities that owns production factors) as well as the keeping track of their specific contribution to gross fixed capital formation (GFCF). In a standard CGE model the representative household, endowed with all production factors, receives total factors income as a global transfer. Furthermore its total GFCF - effective total investment - is generally deduced from the difference between its total savings and the capital balance. In this formulation, public administrations economic transactions are reduced to tax collection, social transfers and final services consumption.

3.3.1 Households (*HH*)

The gross primary income or revenue before tax (RBT_h) of the representative household corresponds to the sum of the elements of the *HH* row of the SAM:

$$RBT_h = \sum_{i=1}^{n} w_i \cdot l_i \cdot Y_i + \overline{\omega_{Kh}} \cdot GOS + \overline{\omega_{LDh}} \cdot LAND + \rho_f \cdot N + \rho_g \cdot N + OT_h \tag{48}$$

The gross primary income is constituted of the following elements:

- Total net income from labor computed as the sum of wages generated by the different economic sectors: $\sum_{i=1}^{n} w_i \cdot l_i \cdot Y_i$

- Total income from capital of households computed as an exogenous share of total *GOS*: $\overline{\omega_{Kh}} \cdot GOS$

- Total land rent earned by households as landlords computed as an exogenous share of total income from land *LAND*: $\overline{\omega_{LDh}} \cdot LAND$

- Social transfers as the sum of two aggregates: social transfers from private source (like private care insurances) and social transfers from public source. The computation is based on the product of an exogenous level of transfer per capita and the total population. Finally: $R_{fh} + R_{gh} = \rho_f \cdot N + \rho_g \cdot N$

- A residual level of transfers computed as an exogenous share of a global pool of others transfers: $OT_h = \overline{\omega_{OTh}} \cdot OT$

The gross disposable income R_h of the representative household is obtained by subtracting from RBT_h the tax on income TI_h levied at a constant average rate on RBT_h minus transfers (Equation 59), and two others direct taxes T_{hf} and T_{hg} (respectively paid to *FIRMS* and *GOV*) that are indexed on GDP:

$$R_h = RBT_h - TI_h - T_{hf} - T_{hg} \tag{49}$$

R_{CONS}, the income expensed in consumption goods is inferred from disposable income by subtracting savings. The savings rate $\overline{\tau_S}$ is exogenous in the model.

$$R_{CONS} = (1 - \overline{\tau_S}) \cdot R_h \tag{50}$$

Furthermore the model specifies the share of savings directly invested in gross capital formation as an exogenous value calibrated at base year:

$$\frac{GFCF_h}{R_h} = \frac{GFCF_{h_0}}{R_{h_0}} \tag{51}$$

Eventually the net financing capacity of *HH* is the remainder of its income:

$$FCAP_h = R_h - R_{CONS} - GFCF_h \tag{52}$$

HH can be creditor or debtor on capital and financial markets if $FCAP_h$ is positive or negative.

3.3.2 Firms (*FIRM*)

The gross primary income (RBT_f) of the representative firm corresponds to the sum of the elements of the *FIRMS* row of the SAM:

$$RBT_f = \overline{\omega_{Kf}} \cdot GOS + \overline{\omega_{LDf}} \cdot LAND + TL_f + T_{hf} + OT_f \tag{53}$$

It is constituted of the following terms:

- Total income from capital (capital equity) earned by firms computed as an exogenous share of total *GOS*: $\overline{\omega_{Kf}} \cdot GOS$
- Total land rent earned by firms computed as an exogenous share of total income from land *LAND*: $\overline{\omega_{LDf}} \cdot LAND$
- A part of payroll taxes addressed to private financial institutions to finance private care and linked to the social transfers R_{fh}. TL_f is levied with an exogenous tax rate on wages: $TL_f = \sum_i \overline{\tau_{Lfi}} \cdot w_i \cdot l_i \cdot Y_i$
- A residual tax levied on households and proportional to labor income: $T_{hf} = \overline{\tau_{hf}} \cdot RL$
- A residual level of transfers computed as an exogenous share of the global pool of others transfers: $OT_f = \overline{\omega_{OTf}} \cdot OT$

The gross disposable income R_f of *FIRMS* is obtained by subtracting from RBT_f total corporate tax TI_f levied at a constant average rate (Equation 59), and the private transfers directed to *HH*, R_{fh}.

$$R_f = RBT_f - R_{fh} - TI_f \tag{54}$$

Eventually the specific gross capital formation of firms ($GFCF_h$) which represents the bulk of investment is given by the rest of the model and especially constrained by the domestic investment balance (see Equation 85)

As for *HH*, the total financing capacity of *FIRMS* is the remainder of the income:

$$FCAP_f = R_f - GFCF_f \tag{55}$$

3.3.3 Public administrations (*GOV*)

Taxes and social security contributions form the larger share of government resources. We distinguishes: payroll taxes (TL_g), taxes on production (T_Y), a remainder of tax levied on *HH* (T_{hg}), taxes on *HH* income (TI_h), taxes on *FIRMS* profits (TI_f) and sales taxes (T_{CONS}) levied on intermediate and final consumption:

$$TL_g = \sum_i \overline{\tau_{Lgi}} \cdot w_i \cdot l_i \cdot Y_i \tag{56}$$

$$T_Y = \sum_i \overline{\tau_{Yi}} \cdot p_{Yi} \cdot Y_i \tag{57}$$

$$T_{hg} = \overline{\tau_{hg}} \cdot RL \tag{58}$$

$$TI_h = \overline{\tau_{Ih}} \cdot (RL + RK_h + \Pi_h + M_h + RLD_h) \tag{59}$$

$$TI_f = \overline{\tau_{If}} \cdot \left(RK_f + \Pi_f + M_f + RLD_f\right) \tag{60}$$

$$T_{CONS} = \sum_i \left(\sum_j p_{BTICij} \cdot \overline{\tau_{CONSi}} \cdot \alpha_{ij} \cdot Y_j \right) + \overline{\tau_{CONSi}} \cdot (p_{BTCi} \cdot C_i + p_{BTGi} \cdot G_i + p_{BTIi} \cdot I_i) \tag{61}$$

The total tax income T is thus given by:

$$T = TL_g + T_Y + T_{CONS} + T_{hg} + TI_h + TI_f + T_{Carb} \tag{62}$$

Furthermore, public administration income balance follows the *GOV* row of the SAM:

$$R_g = T + \overline{\omega_{Kg}} \cdot GOS + \overline{\omega_{LDg}} \cdot LAND + OT_g - R_{gh} \tag{63}$$

Total available income R_g is thus defined as the sum of:

- Total income from taxation:T
- Total income from capital endowment of public companies computed as an exogenous share of total *GOS*: $\overline{\omega_{Kg}} \cdot GOS$
- Total land rent earned from public land computed as an exogenous share of total income from land *LAND*: $\overline{\omega_{LDg}} \cdot LAND$
- A residual level of transfers computed as an exogenous share of the global pool of others transfers:$OT_g = \overline{\omega_{OTg}} \cdot OT$
- A negative contribution linked to the social transfers for *HH*: $-R_{gh} = -\rho_g \cdot N$

Public expenditures are assumed to be indexed on national income, and therefore represent a constant share of GDP:

$$\frac{p_{COMP} \cdot G_{COMP}}{GDP} = \frac{p_{COMP_0} \cdot G_{COMP_0}}{GDP_0} \tag{64}$$

Both private and public social transfers per capita follow GDP per capita:

$$\frac{\rho_g}{\rho_{g_0}} = \frac{\rho_f}{\rho_{f_0}} = \frac{GDP \cdot N_0}{GDP_0 \cdot N} \tag{65}$$

As for public expenditure, public gross capital formation represents a constant share of GDP:

$$\frac{GFCF_f}{GDP} = \frac{GFCF_{f_0}}{GDP_0} \tag{66}$$

Finally $FCAP_g$, the financial capacity of GOV is given:

$$FCAP_g = R_g - p_{COMP} \cdot G_{COMP} - GFCF_g \tag{67}$$

3.3.4 Rest of the World (*ROW*)

The closure of the model is made through the balance of capital flows between the three domestic institutional sectors and the rest of the world:

$$FCAP_m = -\left(FCAP_h + FCAP_f + FCAP_g\right) \tag{68}$$

The sum of Equations 68 and 85 gives the savings-investment balance of the model.

According to Walras law, the last accounting balance, the balance of payments, which balances the *ROW* budget, is given as a linear combination of the others equations of the model:

$$FCAP_m = \sum_i p_{Mi} \cdot M_i - \sum_i p_{Xi} \cdot X_i - \left(OT_h + OT_f + OT_g\right)$$

However, contrary to most CGE models, foreign savings $FCAP_m$ or alternatively the exchanged rate can not be freely fixed in the model. Usually in CGE models, either foreign savings are fixed and the exchanged rate adjusts or else the modeler sets an exogenous exchanged rate and foreign savings are endogenous. In fact in IMACLIM-BR foreign savings are already given by Equation 68 and the specific capital-investment balance (see section 3.7.3). We could alternatively free such specification (like Equation 66 for instance) and set an exogenous trade balance or exchange rate. Eventually capital flows from and to the *ROW* are not assigned a specific behaviour, adapt to the domestic financing needs but constrain the trade balance and the exchanged rate.

At last, as previously mentioned, other transfers OT ("other current transfers") and "capital transfers" are defined as a fixed share of GDP. Other transfers include for example interests payments.

$$\frac{OT}{GDP} = \frac{OT_0}{GDP_0} \tag{69}$$

3.4 Growth engine

As introduced earlier, IMACLIM-BR projects the Brazilian economy in the medium to long run in a single step projection and relies on the method of comparative statics.

In IMACLIM-BR the growth engine is basically exogenous and technical progress is implemented through factor augmenting coefficients.

The growth engine is the combination of several drivers:

- The total population and active population growth:

$$N = (1 + \delta_N)^t \cdot N_0 \tag{70}$$

$$NS = (1 + \delta_{NS})^t \cdot N_0 \tag{71}$$

- The implicit capital accumulation computed through a proportional link between total fixed capital consumption and the current level of total investment in capital good (see eq. (84))
- A Harrod neutral exogenous technical progress on labor, implemented by means of a factor augmenting coefficient (see section 3.5.2)

3.5 Production and final consumption trade-offs

3.5.1 Final consumption trade-offs

Final demand is derived from a utility function of the Stone-Geary form (or Linear expenditure system - LES):

$$U = \prod_i \left(C_i - C_{i_{min}}\right)^{\overline{\alpha_i}} \tag{72}$$

With a LES utility system, only the consumption of goods and services above the basic need level $C_{i_{min}}$ provides utility so that it represents a minimum of consumption that should be satisfied. The marshallian demand of goods and services derived from the maximisation of utility under expensed income constraint R_{CONS} is given by the following equation:

$$\forall i \qquad C_i = C_{i_{min}} + \frac{\overline{\alpha_i}}{p_{Ci}} \cdot \left(R_{CONS} - \sum_j p_{Cj} \cdot C_{j_{min}} \right) \tag{73}$$

The basic needs are described as volumes per capita multiplied by population and are calibrated at base year with the exogenous parameters sh_i:

$$C_{i_{min}} = sh_i \cdot C_{i_0} \cdot \frac{N}{N_0}$$

$\overline{\alpha_i}$ is the share of income (basic needs excluded) devoted to consumption of good i beyond basic need. This constant parameter is calibrated at base year.

3.5.2 Production trade-offs

The structure of production trade-offs are inspired from Ghersi et al. (2011) and the idea that these trade-offs are limited by technical asymptotes that constrain the unit consumptions of factors above some floor values. The assumption is made that the *variable* shares of the unit consumptions of production inputs and factors are substitutable according to a CES specification. The existence of a fix share of each of these consumptions implies that the elasticities of substitution of *total* unit consumptions (sum of the fix and variable shares) are not fixed, but decrease as the consumptions approach their asymptotes. In the meantime, asymptotes make it possible to calibrate specific elasticities of substitution for the different inputs and factors. This provides a convenient way to create simple reduced-forms of bottom-up models.

Under these assumptions and constraints, the formulation of the unitary consumptions of secondary factors α_{ij}, of labour l_j and of capital k_j can be written as the sum of the floor value and a consumption above this value. The latter corresponds to the familiar expression of conditional factor demands of a CES production function with an elasticity of σ_j (the coefficients of which are calibrated at base year).

$$\alpha_{ij} = \frac{1}{\left(1+\psi_{ij}\right)^t} \cdot$$

$$\left[\beta_{ij} \cdot \alpha_{ij0} + \left(\frac{\overline{\lambda_{ij}}}{p_{CIij}}\right)^{\sigma_j} \cdot \left(\sum_{i=1}^{n} \overline{\lambda_{ij}}^{\sigma_j} \cdot p_{CIij}{}^{1-\sigma_j} + \overline{\lambda_{Lj}}^{\sigma_j} \cdot \frac{p_{Lj}}{\left(1+\phi_{Lj}\right)^t}^{1-\sigma_j} + \overline{\lambda_{Kj}}^{\sigma_j} \cdot p_K{}^{1-\sigma_j} + \overline{\lambda_{LDj}}^{\sigma_j} \cdot p_{LDj}{}^{1-\sigma_j}\right)^{\frac{\sigma_j}{1-\sigma_j}}\right] \tag{74}$$

$$l_j = \frac{1}{\left(1+\phi_{Lj}\right)^t} \cdot$$

$$\left[\beta_{Lj} \cdot l_{j0} + \left(\frac{\overline{\lambda_{Lj}}}{\frac{p_{Lj}}{\left(1+\phi_{Lj}\right)^t}}\right)^{\sigma_j} \cdot \left(\sum_{i=1}^{n} \overline{\lambda_{ij}}^{\sigma_j} \cdot p_{CIij}{}^{1-\sigma_j} + \overline{\lambda_{Lj}}^{\sigma_j} \cdot \frac{p_{Lj}}{\left(1+\phi_{Lj}\right)^t}^{1-\sigma_j} + \overline{\lambda_{Kj}}^{\sigma_j} \cdot p_K{}^{1-\sigma_j} + \overline{\lambda_{LDj}}^{\sigma_j} \cdot p_{LDj}{}^{1-\sigma_j}\right)^{\frac{\sigma_j}{1-\sigma_j}}\right] \tag{75}$$

$$k_j = \frac{1}{\left(1+\psi_{Kj}\right)^t} \cdot$$

$$\left[\beta_{Kj} \cdot k_{j0} + \left(\frac{\overline{\lambda_{Kj}}}{p_K}\right)^{\sigma_j} \left(\sum_{i=1}^{n} \overline{\lambda_{ij}}^{\sigma_j} \cdot p_{CIij}{}^{1-\sigma_j} + \overline{\lambda_{Lj}}^{\sigma_j} \cdot \frac{p_{Lj}}{\left(1+\phi_{Lj}\right)^t}^{1-\sigma_j} + \overline{\lambda_{Kj}}^{\sigma_j} \cdot p_K{}^{1-\sigma_j} + \overline{\lambda_{LDj}}^{\sigma_j} \cdot p_{LDj}{}^{1-\sigma_j}\right)^{\frac{\sigma_j}{1-\sigma_j}}\right] \tag{76}$$

$$ld_j = \frac{1}{\left(1+\psi_{LDj}\right)^t} \cdot$$

$$\left[\beta_{LDj} \cdot ld_{j0} + \left(\frac{\overline{\lambda_{LDj}}}{p_{LDj}}\right)^{\sigma_j} \cdot \left(\sum_{i=1}^{n} \overline{\lambda_{ij}}^{\sigma_j} \cdot p_{CIij}{}^{1-\sigma_j} + \overline{\lambda_{Lj}}^{\sigma_j} \cdot \frac{p_{Lj}}{\left(1+\phi_{Lj}\right)^t}^{1-\sigma_j} + \overline{\lambda_{Kj}}^{\sigma_j} \cdot p_K{}^{1-\sigma_j} + \overline{\lambda_{LDj}}^{\sigma_j} \cdot p_{LDj}{}^{1-\sigma_j}\right)^{\frac{\sigma_j}{1-\sigma_j}}\right] \tag{77}$$

This sum is however modified to take into account the combination of exogenous labor productivity improvements ϕ_{Lj} - implemented as factor augmenting productivity gains - and structural changes for input and factor consumption, implemented as simple multipliers $\frac{1}{(1+\psi_{ij})^t}$ or $\frac{1}{(1+\psi_{Kj})^t}$. These multipliers makes it possible to drive changes in production patterns in order to mimic specific energy and economic scenarios in terms of trend energy intensity of production, fuel mixes, capital intensity, etc.

In addition, let us emphasize again that the "cost of capital" p_K entering the production trade-offs is *stricto sensu* the price of "machine capital", i.e. equal to a simple weighted sum of the investment prices of immobilized goods (eq. (43)), which is not directly related to the actual return on capital. It is possible to track returns on capital *ex post* based on investments levels and gross operating surplus.

3.6 Trade

The competition on international markets relies first of all on relative prices. The ratio of imports to domestic production on the one hand, and the "absolute" exported quantities on the other hand, are elastic to the terms of trade, according to constant, product-specific elasticities. Three specifics are added to this standard framework. Energy imports are treated specifically and imports trends outside terms of trade are driven by sh_{Mi} multipliers to mimic given energy scenarios. Second, non-energy imports are treated as "income elastic" beyond terms of trade, which is implemented with constant elasticites to GDP level. This feature reflects the trend towards increasing shares of imports as GDP grows, which has been observed in Brazil in the last decades (dos Santos et al., 2011). Eventually, exports are impacted by global economic growth, independently of terms of trade variations. This is captured by assuming an exogenous rate of growth of exports δ_{Xi}.

Energy goods:

$$\frac{M_i}{Y_i} = sh_{Mi} \cdot \frac{M_{i_0}}{Y_{i_0}} \cdot \left(\frac{p_{Mi_0} \cdot p_{Yi}}{p_{Yi_0} \cdot p_{Mi}}\right)^{\sigma_{Mi}} \tag{78}$$

Non energy goods:

$$\frac{M_i}{Y_i} = \frac{M_{i_0}}{Y_{i_0}} \cdot \left(\frac{p_{Mi_0} \cdot p_{Yi}}{p_{Yi_0} \cdot p_{Mi}}\right)^{\sigma_{Mi}} \cdot \left(\frac{GDP}{GDP_0}\right)^{(\sigma_{MYi}-1)} \tag{79}$$

All goods:

$$\frac{X_i}{X_{i_0}} = \left(\frac{p_{Mi_0} \cdot p_{Xi}}{p_{Xi_0} \cdot p_{Mi}}\right)^{\sigma_{Xi}} \cdot (1+\delta_{Xi})^t \tag{80}$$

3.7 Market and accounting balances

3.7.1 Goods markets

Goods market clearing is a simple accounting balance between resources (production and imports) and uses (households and public administrations' consumption, investment, exports). Thanks to the process of hybridization, this equation is written in Mtoe for energy goods and consistent with the 2005 Brazilian energy balance (notwithstanding that the G and I of energy goods are nil by definition).

$$\forall i \in \{1, ..., n\} \qquad Y_i + M_i = \sum_{j=1}^{n} \alpha_{ij} \cdot Y_j + C_i + G_i + I_i + X_i \tag{81}$$

3.7.2 Labor market

The labor market conditions, results from the interplay between labor demand from the production systems, equal to the sum of their factor demands $l_i \cdot Y_i$, and of labor supply from households. As part of key structural assumptions, the model allows for a strictly positive unemployment rate u and the market balance writes:

$$\sum_{i=1}^{n} l_i \cdot Y_i = (1-u) \cdot NS \tag{82}$$

The unemployment level depends on a so-called "wage curve" (Blanchflower and Oswald, 1995) which synthesizes the forces that drive average wage formation. Such an empirical curve can be interpreted as the result of wage bargain between employers and employees or as an aggregate labor supply curve. Within the wage curve, indexation can vary from pure nominal to pure real wage indexation according to the coefficient sh_r.

$$\frac{w}{\left(1+\phi_{Lj}\right) \cdot (sh_r \cdot CPI + (1-sh_r)) \cdot w_0} = \left(\frac{u}{u_{ref}}\right)^{\sigma_u} \tag{83}$$

This makes it possible to test alternative visions of labor markets and their consequences on policy analysis. As studied by Thubin (2012), using a wage curve enables to cover the key debatable issues about labor markets and their consequences on carbon tax policy assessment. Our wage curve depends on two key parameters: the elasticity σ_u and the coefficient of indexation sh_r. Within the labor supply interpretation of the wage curve, the case where $\sigma_u = -\infty$ implies a fixed level of employment, which can be interpreted as a fixed level of labor supply. In this case the average wage is perfectly flexible and labor market behaves as a perfect market. σ_u further enables to test variant levels of wage rigidity in the long run. In addition, the wage curve can be more or less indexed on nominal or real wages. The indexation on real wages reflects a force centered on the purchasing power of labor income. Conversely, nominal wage indexation reflects a higher international competition between national and foreign workers, so that wages are compared to the international price index (the *numéraire* of the model.

Let us remind that a real metric of labor is used in the model - full time equivalent jobs - and sector specific wages. We make the assumption that in the long run workers are substitutable independently from the sector and the respective labor productivity.

3.7.3 Investment and capital flows

Contrary to standard CGE models, IMACLIM-BR does not represent explicit capital markets and the capital-investment balance is "demand-driven". As previously highlighted, productive sectors arbitrate capital consumption according to prices of equipment. Then total capital consumption translates into a demand for investment at time t through the β vector within the implicit formulation of smooth capital accumulation.

$$\frac{I_i}{\sum_{j=1}^{n} k_j \cdot Y_j} = \overline{\beta_i} = \frac{I_{i_0}}{\sum_{j=1}^{n} k_{j_0} \cdot Y_{j_0}} \tag{84}$$

In the meantime the assumption is made of a single investment good in the economy as a weighted sum of different goods calibrated at base year (β vector).

Furthermore, the supply for investment adapts to the demand and capital formation from firms complete households and public contribution to satisfy that demand and balance investment flows.

$$GFCF_h + GFCF_f + GFCF_g = \sum_{i=1}^{n} p_{Ii} \cdot I_i \tag{85}$$

Eventually, the investment balance, together with households saving rate and public expenses, imposes the external or trade balance which is endogenous in the model.

3.7.4 Land market

In this version of the model, the price of land is exogenous and follows the exogenous increase of land productivity:

$$p_{LDi} = p_{LDi_0} \cdot \left(1 + \phi_{LDi}\right) \tag{86}$$

In others versions of IMACLIM-BR , specific constraints on land allocation are implemented to study land-use issues.

3.8 Carbon tax policy

The model is specifically designed to study carbon tax policies in the medium to long run by generating policy-constrained projections. In the model, implementing a carbon tax amounts to adding a shock on fossil fuels prices proportional to their carbon content at the time horizon studied. Within our one-step projection framework, the underlying assumption is that a phase-in carbon tax is applied in the economy starting, say in 2015, with a small level to reach the ultimate carbon tax level at the time t studied. Accordingly, the model represents the result of technico-economic adjustments and market interactions at t as the end of a smooth pathway which undergoes a rising carbon tax.

The policy constrained price system is the following - with t_{Carb} the carbon price and $\overline{\gamma_{ij}}$ the emission factor of energy good i for sector j:

$$p_{ICij} = p_i \cdot \left(1 + \tau_{BMi} + \tau_{TMi} + \overline{\tau_{MSICij}}\right) \cdot (1 + \overline{\tau_{CONSi}}) + t_{Carbj} \cdot \overline{\gamma_{ij}} \tag{87}$$

$$p_{Ci} = p_i \cdot (1 + \tau_{BMi} + \tau_{TMi} + \overline{\tau_{MSCi}}) \cdot (1 + \overline{\tau_{CONSi}}) + t_{Carbh} \cdot \overline{\gamma_{ih}} \tag{88}$$

This framework can accommodate sector specific carbon prices.

Total carbon revenues are the sum of carbon taxes levied on the different sectors:

$$T_{Carb} = \sum_i \left(\sum_j t_{Carbj} \cdot \overline{\gamma_{ij}} \cdot \alpha_{ij} \cdot Y_j \right) + \sum_i t_{Carbh} \cdot \overline{\gamma_{ih}} \cdot C_i \tag{89}$$

In the present model version, IMACLIM-BR can simulate four different carbon tax policies based on four different carbon revenues recycling options:

- No recyling (NR): carbon revenues feed public budget

- Reduction of sales taxes (RST): this recycling option consists in reducing the sector specific rates of sales taxes $\overline{\tau_{CONSi}}$ by the same coefficient δ_{CONS} for all sectors while maintaining neutral policy budget:

$$\forall i \in \{1, ..., n\} \qquad \tau_{CONSi} = \overline{\tau_{CONSi}} \cdot (1 - \delta_{CONS}) \tag{90}$$

$$T_{Carb} = \delta_{CONS} \sum_i \left(\sum_j p_{BTICij} \cdot \overline{\tau_{CONSi}} \cdot \alpha_{ij} \cdot Y_j \right) + \overline{\tau_{CONSi}} \cdot (p_{BTCi} \cdot C_i + p_{BTGi} \cdot G_i + p_{BTIi} \cdot I_i) \tag{91}$$

- Reduction of payroll taxes (RPT): this recycling option consists in reducing the sector specific rates of payroll taxes $\overline{\tau_{Lgi}}$ by the same coefficient δ_{Lg} for all sectors while maintaining neutral policy budget:

$$\forall i \in \{1, ..., n\} \qquad \tau_{Lgi} = \overline{\tau_{Lgi}} \cdot \left(1 - \delta_{Lg}\right) \tag{92}$$

$$T_{Carb} = \delta_{Lg} \cdot \left(\sum_i \overline{\tau_{Lgi}} \cdot w_i \cdot l_i \cdot Y_i \right) \tag{93}$$

- Lump sum transfer to households (LS): total carbon revenues are directly transferred to households while maintaining neutral policy budget:

$$R_{gh} = \rho_g \cdot N + Lump \tag{94}$$

$$T_{Carb} = Lump \tag{95}$$

4 Two expanded model versions

4.1 A 6-sectors version with 6 households groups - Imaclim-BR 6-6

The Imaclim-BR 6-6 version contemplates the desegregation of the *Households* institutional sector in six sub-groups defined by income brackets at base year. The behavior of each group or class is synthesized by means of a representative households of that group. In this version of the model, a few equations are modified: the equation of households accounts and the final consumption trade-off. The gross primary income of class h is:

$$RBT_h = \overline{\omega_{Lh}} \cdot \left(\sum_{i=1}^{n} w_i \cdot l_i \cdot Y_i \right) + \overline{\omega_{Kh}} \cdot (\overline{\omega_{KH}} \cdot GOS) + \overline{\omega_{LDh}} \, (\overline{\omega_{LDH}} \cdot LAND) + \rho_{fh} \cdot N_h + \rho_{gh} \cdot N_h + OT_h \tag{96}$$

It is constituted of the following elements:

- A share $\overline{\omega_{Lh}}$ of total net income from labor calibrated at base year: $\overline{\omega_{Lh}} \cdot \left(\sum_{i=1}^{n} w_i \cdot l_i \cdot Y_i \right)$. For specific scenarios, these shares can be modified to reflect changes in labor income distribution patterns among groups.
- A share $\overline{\omega_{Kh}}$ of total income from capital of households: $\overline{\omega_{Kh}} \cdot (\overline{\omega_{KH}} \cdot GOS)$.
- A share $\overline{\omega_{LDh}}$ of total land rent earned by households as landlords: $\overline{\omega_{LDh}} \, (\overline{\omega_{LDH}} \cdot LAND)$.
- Social transfers as the sum of two aggregates: social transfers from private source (like private care insurances) and social transfers from public source. The computation is based on the product of an exogenous level of transfer per capita calibrated at base year and the population of class h. Finally: $R_{fh} + R_{gh} = \rho_{fh} \cdot N_h + \rho_{gh} \cdot N_h$
- A residual level of transfers computed as an exogenous share of a global pool of others transfers to households: $OT_h = \overline{\omega_{OTh}} \cdot OTH$

The structure of expenses is similar to the single class model, including tax rates, saving rates and rates of capital formation class specific and calibrated at base year.

As far as consumption trade-offs are concerned, the 6-6 model version includes class specific LES expenditure systems.

4.2 A 12-sectors version with an expanded demand system and a specific oil sector - IMACLIM-BR 12-ext

4.2.1 An expanded demand system

In the IMACLIM-BR 12-ext version, the utility of households depends on four types of goods and services. These four goods and services are (i) *food goods (FD)* , (ii) *transport services (TRA)*, (iii) *residential energy services (RES)* and (iv) a composite nest of *others goods and services (OTH)*, which correspond to the rest of manufactured goods and services. The first layer of the utility function is a LES utility:

Sectors		
Production	**HH consumption**	**Production factors**
Energy		
Bioenergy (BIO)	Biofuels (BIOTRA) Firewood (BIORES)	Labor Capital Land
Coal (COAL)		
Crude Oil (OIL)		
Natural Gas (GAS)	-	
Refined Oil (ROIL)	Oil fuels (ROILTRA) LPG (ROILRES)	
Electricity (ELEC)	-	
Non-Energy		
Load Transportation (LOAD)	-	
Pass. Transportation (PASS)	-	
Livestock (CATT)	-	
Agro-industry (AGRI)	-	
Industry (INDUS)	Transport vehicles (INDVHC) Others manufactured goods (INDOTH)	
Services (COMP)	-	

Table 3: Sectors, consumption goods and factors in IMACLIM-BR 12-ext version

$$U = (C_{FD} - C_{FD_{min}})^{\overline{\alpha_{FD}}} \cdot \left(C_{TRA} - C_{TRA_{min}}\right)^{\overline{\alpha_{TRA}}} \cdot \left(C_{RES} - C_{RES_{min}}\right)^{\overline{\alpha_{RES}}} \cdot \left(C_{OTH} - C_{OTH_{min}}\right)^{\overline{\alpha_{OTH}}} \quad (97)$$

The marshallian demands of goods and services derived from this utility system, are:

$$\forall i \in [FD, TRA, RES, OTH] \qquad C_i = C_{i_{min}} + \frac{\overline{\alpha_i}}{p_{Ci}} \cdot \left(R_{CONS} - \sum_j p_{Cj} \cdot \overline{C_j}\right) \quad (98)$$

The basic needs are described as volumes per capita multiplied by population and are calibrated at base year with the exogenous parameters sh_i:

$$C_{i_{min}} = sh_i \cdot C_{i_0} \cdot \frac{N}{N_0}$$

The strength of such a demand system is to base households utility on the goods and services that really are at play in preferences trade-offs. This further makes it possible to detail the chain of underlying goods and services consumption that ultimately serve to satisfy these services demands. Basically, the purchase of private vehicles and fuel consumption can only be thought as combined to satisfy transportation needs. The expanded demand system contemplated is represented in figure 3.

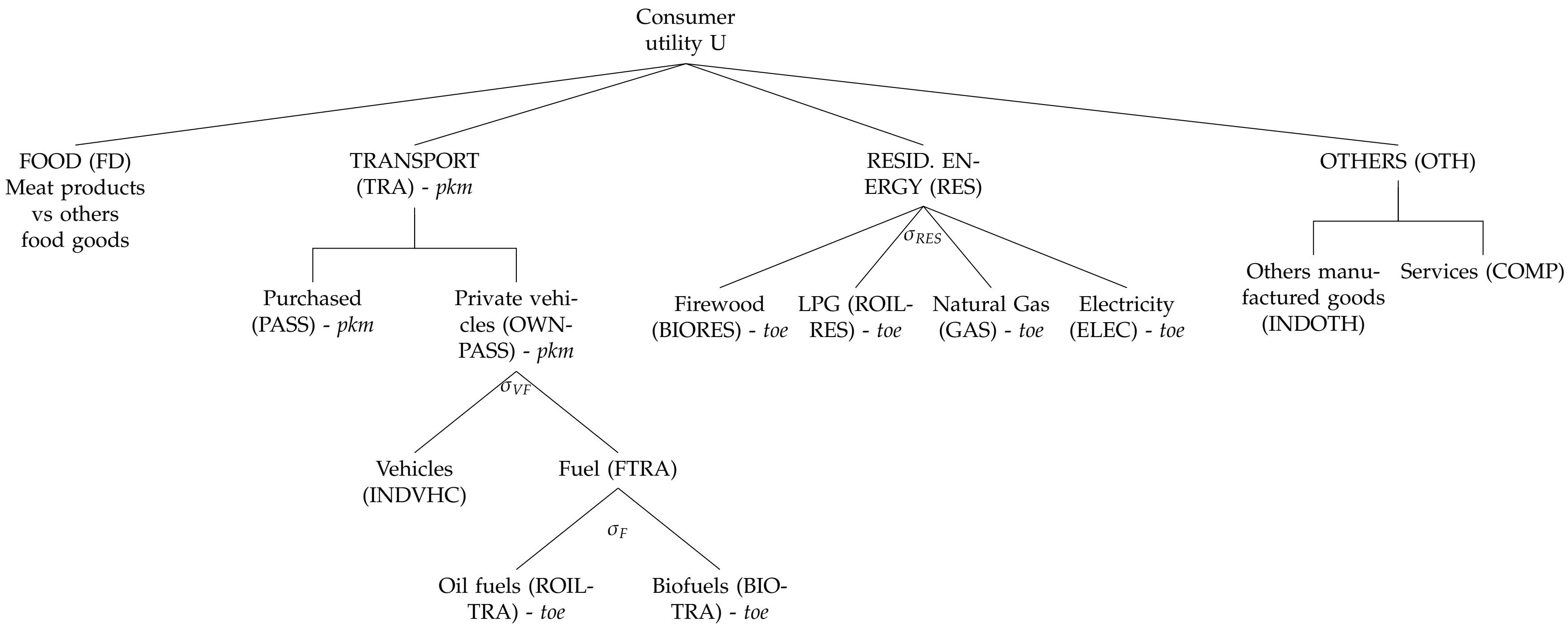

Figure 3: Imaclim-BR 12-ext demand system

Down-stream nests correspond to either CES or Leontief specifications calibrated at base year.

Food is constituted of beef products (Cattle) and other food goods that substitute with the elasticity σ_{MR}. This elasticity controls the diet changes according to price levels. Residential energy sources substitutions are controlled by σ_{HOUS}. The dual accounting structure of the model and the accounting of energy flows in physical units offers room to fine-tune the projection of energy demand structure. In particular we choose to implement exogenous trends in residential energy use structure to stick to bottom-up projections by means of exogenous evolution rates of energy sources shares outside relative prices change. This is a simple and convenient way to compensate for the expense-share conservative properties of CES nests in order to implement structural changes in energy consumption[4]. This is especially important to generate long run projections for emerging economies. The dual accounting schemes makes it possible to ground these structural changes on bottom-up projections. To give just one example for Brazil, as average income grows, households are expected to massively get rid of traditional biomass to turn to modern energy sources like gas and grid electricity.

Demand levels linked to Residential energy bundle are given as follows:

$$\forall i \in [BIORES, ROILRES, GAS, ELEC] \qquad C_i = \frac{C_{RES}}{1+\delta_{RESi}} \cdot \overline{\lambda_i}^{\sigma_{RES}} \cdot \left(\frac{p_{C_{RES}}}{\frac{p_{Ci}}{1+\delta_{RESi}}} \right)^{\sigma_{RES}} \tag{99}$$

$$p_{C_{RES}} = \left(\sum_i \overline{\lambda_i}^{\sigma_{RES}} \cdot \frac{p_{Ci}}{1+\delta_{RESi}}^{1-\sigma_{RES}} \right)^{\frac{1}{1-\sigma_{RES}}} \tag{100}$$

The transportation bundle is more expanded. It starts with the breakdown between purchased transportation services (public transportation, taxis, etc.) and own transportation supplied by private light duty vehicles. In the model the breakdown is Leontief, which implies it is a structural feature of future paths:

$$\forall i \in [PASS, OWNPASS] \qquad C_i = C_{TRA} \cdot \left(\frac{C_{i_0}}{C_{TRA_0}} \right) \tag{101}$$

The own transportation bundle is a CES nest of private vehicles purchase and fuel

[4]It is to be noted that these coefficients are not AEEI (Autonomous energy efficiency index) coefficients and do not incur so-called "rebound effect". They are just multipliers to implement specific trend structural changes in energy consumption

consumption:

$$C_{INDVHC} = C_{OWNPASS} \cdot \overline{\lambda_{INDVHC}}^{\sigma_{VF}} \cdot \left(\frac{p_{C_{OWNPASS}}}{p_{C_{INDUS}}}\right)^{\sigma_{VF}} \tag{102}$$

$$C_{FTRA} = \frac{C_{OWNPASS}}{1 + \delta_{EffFuel_t}} \cdot \overline{\lambda_{FTRA}}^{\sigma_{VF}} \cdot \left(\frac{p_{C_{OWNPASS}}}{\frac{p_{C_{FTRA}}}{1+\delta_{EffFuel_t}}}\right)^{\sigma_{VF}} \tag{103}$$

$$p_{C_{OWNPASS}} = \left(\overline{\lambda_{INDVHC}}^{\sigma_{VF}} \cdot p_{C_{INDUS}}{}^{1-\sigma_{VF}} + \overline{\lambda_{FTRA}}^{\sigma_{VF}} \cdot \frac{p_{C_{FTRA}}}{1 + \delta_{EffFuel_t}}{}^{1-\sigma_{VF}}\right)^{\frac{1}{1-\sigma_{VF}}} \tag{104}$$

$\delta_{EffFuel_t}$ is a coefficient of autonomous efficiency gains (AEEI) of private vehicles (light duty vehicles mainly).

The fuel bundle is a CES nest of biofuels (ethanol) and oil fuels (gasoline mainly and diesel):

$$C_{BIOTRA} = C_{FTRA} \cdot \overline{\lambda_{BIOTRA}}^{\sigma_F} \cdot \left(\frac{p_{C_{FTRA}}}{p_{C_{BIOTRA}}}\right)^{\sigma_F} \tag{105}$$

$$C_{ROILTRA} = C_{FTRA} \cdot \overline{\lambda_{ROILTRA}}^{\sigma_F} \cdot \left(\frac{p_{C_{FTRA}}}{p_{C_{ROILTRA}}}\right)^{\sigma_F} \tag{106}$$

$$p_{C_{FTRA}} = \left(\overline{\lambda_{BIOTRA}}^{\sigma_F} \cdot p_{C_{BIOTRA}}{}^{1-\sigma_F} + \overline{\lambda_{ROILTRA}}^{\sigma_F} \cdot p_{C_{ROILTRA}}{}^{1-\sigma_F}\right)^{\frac{1}{1-\sigma_F}} \tag{107}$$

Eventually, the bundle for Others goods is Leontief:

$$\forall i \in [INDOTH, COMP] \qquad C_i = C_{OTH} \cdot \left(\frac{C_{i_0}}{C_{OTH_0}}\right) \tag{108}$$

4.3 Oil and liquid fuels sectors

The 12-ext model version makes it possible to assess the articulation between oil and climate policies. In particular, the models enables to test various targets of crude oil production in the medium run in order to assess its impacts on the economy and growth. Accordingly, this model version sets the level of domestic crude oil production as an exogenous parameter whereas it is endogenous in the reference version. Domestic oil consumption is endogenous and depends on market mechanisms so that oil exports represent the reminder of oil production. Such a framework makes it possible to endogenize the breakdown of oil production (between domestic consumption and exports) according to market conditions - on liquid fuels especially - for a given level production. In practice the equation for crude oil exports is replaced by the following equation which sets the level of domestic production at time t:

$$Y_{OIL} = Y_{OIL_0} \cdot (1 + \delta_{YOIL})^t \tag{109}$$

Let us remind, that already in the reference version, the oil price of exports follows the world oil price:

$$p_{X_{OIL}} = \frac{p_{X_{OIL_0}}}{p_{M_{OIL_0}}} \cdot p_{M_{OIL}} \tag{110}$$

Finally, alternative tax conditions about liquid fuels can be tested by playing with the system of sales tax τ_{CONS}. This makes it possible to assess alternative fuel price policies and the competition between biofuels and oil fuels for transportation especially.

4.4 Miscellaneous specifics

4.4.1 Transport margins

Business margins τ_{BMi} and transport margins τ_{TMi}, identical for all intermediate and final consumptions of good i, are calibrated at base year and kept constant - except those on *LOAD* sector and trade activities aggregated within the *COMP* sector, which are simply adjusted, to have the two types of margins sum up to zero :

$$\forall i \notin [COMP, LOAD] \qquad \begin{cases} \tau_{BMi} = \overline{\tau_{BMi}} \\ \tau_{TMi} = \overline{\tau_{TMi}} \end{cases} \tag{111}$$

$$\sum_{j=1}^{n} \tau_{BM_{COMP}} \cdot p_{COMP} \cdot \alpha_{COMPj} \cdot Y_j + \tau_{BM_{COMP}} \cdot p_{COMP} \cdot (C_{COMP} + G_{COMP} + I_{COMP} + X_{COMP}) + \sum_{i \neq COMP} \sum_j \overline{\tau_{BMi}} \cdot p_i \cdot \alpha_{ij} \cdot Y_j + \sum_{i \neq COMP} \overline{\tau_{BMi}} \cdot p_i \cdot (C_i + G_i + I_i + X_i) = 0 \tag{112}$$

$$\sum_{j=1}^{n} \tau_{BM_{LOAD}} \cdot p_{LOAD} \cdot \alpha_{LOADj} \cdot Y_j + \tau_{BM_{LOAD}} \cdot p_{LOAD} \cdot (C_{LOAD} + G_{LOAD} + I_{LOAD} + X_{LOAD}) + \sum_{i \neq LOAD} \sum_j \overline{\tau_{BMi}} \cdot p_i \cdot \alpha_{ij} \cdot Y_j + \sum_{i \neq LOAD} \overline{\tau_{BMi}} \cdot p_i \cdot (C_i + G_i + I_i + X_i) = 0 \tag{113}$$

4.4.2 Agriculture sectors

The sectors that use the specific land factor (*BIO*, *CATT* and *AGRI*) generate a total available land income:

$$LAND = \sum_{i=1}^{n} p_{LDi} \cdot ld_i \cdot Y_i \text{ with } ld_i = 0 \text{ for sectors outside BIO, CATT and AGRI} \quad (114)$$

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Appendix A. Data, variables, parameters and calibration choices for IMACLIM-BR reference settings

A.1 Data: construction of the benchmark hybrid Social Accounting Matrix 2005

The benchmark SAM 2005 is used to calibrate all IMACLIM-BR versions.

A.1.1 Construction of the SAM

National accounting statistics provide a comprehensive numerical framework for macroeconomic simulation exercises. In its current version, IMACLIM BRAZIL is calibrated at base year 2005 on a SAM built on data from two synthesis tables produced by the *Instituto Brasileiro de Geografia e Estatistica* (IBGE):

- The MIP (*Matriz Insumo Produto*, input-output table) balances the uses and resources of products—up to 110 of them in its most disaggregated version.
- The CEI (*Contas Econômicas Integradas*) details the primary and secondary distribution of income between 6 "institutional sectors", i.e. aggregate economic agents: financial firms, non-financial firms, households, non-profit organisations, public administrations, "rest of the world".

Raw MIP data are processed to obtain a description of production and consumption in a square "product - product" system, with no accumulation of stocks. The CEI is aggregated into 4 institutional sectors (households, firms, public administrations and "rest of the world"), and its many entries are simplified into a set of transfers at a level of aggregation comparable to that of the MIP. Basically the primary distribution of income is composed of productive factor remuneration (labor, productive capital and land) and income from property (financial income). The secondary distribution of income is made of indirect taxes and social transfers. Its use ultimately allows extending the traditional framework of general equilibrium modelling to the distribution of national income between economic agents, the resulting changes in the financial positions of those agents, and the corresponding debt payments. MIP and CEI data are finally combined in a unique SAM framework.

A.1.2 Data Hybridization

Considering its focus on climate policy assessment, IMACLIM BRAZIL requires a high degree of realism in the description of the energy inputs to production, and the energy consumptions of households. Indeed, the greenhouse gas emissions targeted by climate policies arise from the consumption of explicit physical quantities that are poorly represented by the quasi-quantities commonly obtained from the MIP through the normalization of output prices. Therefore, a rigorous calibration of the model requires some accurate accounting of the physical quantities of energy consumed, expressed in a relevant unit (e.g. million-tons-of-oil-equivalent, Mtoe).

Such an accounting is found in the national energy balance (*Balanço Energétic National*, BEN) compiled by EPE (*Empresa de Pesquisa Energética*). It is also possible to gather from various sources (BEN, ANE – *Agência National do Petroleo*, etc.) observed final prices for each type of energy, or aggregate thereof, which are indeed end-use specific. The term-by-term product of energy balances and agent-specific prices (the single-price assumption is abandoned) defines a matrix of energy consumptions in monetary terms, which does not match that embedded in the MIP for energy products, for a variety of reasons (the inclusion of services beyond the sheer energy consumptions, the heterogeneity of products, biases from the statistical balancing methods,etc.). Hybridization of the MIP then consists in imputing the differences between the values found in the MIP, and those computed from energy statistics, to some non-energy goods and sectors. For lack of a better hypothesis the value-added of the energy products are corrected pro-rata this imputation. In this way, the product desaggregation is amended, while the cross-sectoral totals are kept consistent with the original statistics. The calibration of the model on this hybrid MIP (which is included in the final benchmark SAM) eventually leads it to depict (i) volumes of the non-energy goods that are traditionally derived from the single-(normalized)-price assumption, and (ii) volumes and prices of the energy goods that are strictly aligned on the available statistics. The differences in price of the same energy good from one agent to the other (e.g. the variable average prices of 1 kWh of electricity) are accounted for by calibrating "specific margins" to the different uses.

The technical details of the hybridization procedure are reported in **??** on the case of France 2010. We follow the same procedure to build the hybrid I-O table for Brazil 2005.

A.1.3 Disaggregation of households in 6 income groups

The disaggregation of the "representative household" in 6 income classes is based on an extrapolation of the Household Budget Survey 2002-2003 (IBGE, 2004), which extensively covers the resources and uses of Brazilian households. The method of disaggregation in 6 income brackets carried-out, is similar to that of Grottera et al.

(2015).

A.2 Variables, parameters and choice of parameters values

The quantitative model implementation distinguishes between model variables, calibrated parameters and non-calibrated parameters. The set of variables characterizes the state of the energy-economic system at the time horizon studied. Calibrated parameters are characteristic of the system and are estimated so that the model reproduces base year SAM. Exogenous non-calibrated parameters are of two types: (i) structural parameters: parameters controlling the functioning of the economic system - elasticity of substitution for instance - and (ii) scenarios parameters - including policy parameters like the carbon tax - used to generate contrasted scenarios.

A.2.1 Variables and parameters in IMACLIM-BR 6 reference setting

Table 4, table 5 and table 6 provide the complete list of variables, calibrated and non-calibrated parameters.

Variable	Description
α_{ij}	Technical coefficient, quantity of input *i* per unit of output *j*
a	Endogenous multiplier for net wages evolution
C_i	Level of *HH* final consumption of good or service *i*, real terms - physical unit for energy (ktoe)
$C_{i_{min}}$	Level of *HH* basic need for good or service *i* in the LES utility function
CPI	Consumer Price Index (Fisher)
δ_{Lg}	Rate of decrease of payroll tax rate in case of related carbon tax recycling scheme
$FCAP_h$	Self-financing capacity of *HH*
$FCAP_f$	Self-financing capacity of *FIRMS*
$FCAP_g$	Self-financing capacity of *GOV*
$FCAP_m$	Self-financing capacity of *ROW*
GDP	Gross Domestic Product
$GFCF_h$	Gross fixed capital formation of *HH*
$GFCF_f$	Gross fixed capital formation of *FIRMS*

$GFCF_g$	Gross fixed capital formation of *GOV*
G_i	Level of *GOV* final consumption of good *i*, real terms
GOS	Gross operating surplus (land rent excluded)
I_i	Level of final consumption for investment in good *i*, real terms
k_j	Capital intensity of sector *j*
l_j	Labor intensity of sector *j*
$LAND$	Total land rent across sectors
ld_j	Land intensity of sector *j*
M_i	Level of imports of good *i*, real term
M_S	Total specific margin - Nil at base year
N	Total population
NS	Total active population
OT	Others transfers
OT_h	Others transfers to *HH*
OT_f	Others transfers to *FIRMS*
OT_g	Others transfers to *GOV*
p_i	Average price of *i*
p_{BTICij}	Before tax purchaser's price of good *i* for sector *j*
p_{BTZi}	Before tax purchaser's price of good *i* for final demand sectors
p_{Ci}	Purchaser's price of good or service *i* for *HH* final consumption
p_{Gi}	Purchaser's price of good *i* for *GOV* final consumption
p_{Ii}	Purchaser's price of good *i* for investment (Gross capital formation)
p_{ICij}	Purchaser's price of good *i* for sector *j*
p_K	Cost of capital input (weighted sum of investment prices)
p_{Lj}	Cost of labor for sector *j*
p_{LDj}	Cost of land for sector *j* (*BIO*, *CATT* and *AGRI* sectors)
p_{Mi}	Import / world price of good *j*

p_{Xi}	Purchaser's price of good i for exports
p_{Yj}	Producer's price of good j
ρ_f	Rate of private transfers per capita
ρ_g	Rate of social transfers per capita
R_h	Gross disposable income of *HH*
R_f	Gross disposable income of *FIRMS*
R_g	Gross disposable income of *GOV*
RBT_h	Revenue before tax / Gross primary income of *HH*
RBT_f	Revenue before tax / Gross primary income of *FIRMS*
R_{CONS}	Consumed income of *HH*
$\tau_{BM_{COMP}}$	Rate of business margin (negative) for *COMP* sector
$\tau_{TM_{COMP}}$	Rate of transport margin (negative) for *COMP* sector
T	Total tax revenues
T_{hf}	Indirect transfers from *HH* to *FIRMS*
T_{hg}	Indirect transfers / tax from *HH* to *GOV*
T_{Carb}	Total carbon taxes
T_{CONS}	Total sales taxes
TI_h	Income tax payments from *HH*
TI_f	Corporate tax revenues
TL_f	Payroll taxes paid to *FIRMS*
TL_g	Payroll taxes paid to *GOV*
TY	Tax on production
u	Rate of unemployment
w_j	Average net wage in the production of sector j
w	Average net wage across sectors
X_i	Level of exports of good i, real term
Y_i	Level of production of good / sector i, real term

Table 4: List of variables

Calibrated parameters	Description
$\overline{\pi_i}$	Mark-up rate (rate of net operating surplus) in the production of good *i*
$\overline{\tau_{Yj}}$	Rate of tax on production
$\overline{\tau_{BMi}}$	Rate of business margin on purchase of good *i*
$\overline{\tau_{TMi}}$	Rate of transport margin on purchase of good *i*
$\overline{\tau_{MSICij}}$	Specific mark-up rate on intermediate consumption of energy good *i*. Defined during the hybridization process
$\overline{\tau_{MSZi}}$	Specific mark-up rate on final consumption of energy good *i*. Defined during the hybridization process
$\overline{\tau_{CONSi}}$	Rate of aggregate sales tax on purchase of good *i*
$\overline{\tau_{Lfi}}$	Rate of payroll taxes paid to private companies by sector *i*
$\overline{\tau_{Lgi}}$	Rate of payroll taxes paid to public administration by sector *i*
$\overline{\omega_{Kh}}$	Share of capital income accruing to *HH*
$\overline{\omega_{Kf}}$	Share of capital income accruing to *FIRMS*
$\overline{\omega_{Kg}}$	Share of capital income accruing to *GOV*
$\overline{\omega_{LDh}}$	Share of land rent accruing to *HH*
$\overline{\omega_{LDf}}$	Share of land rent accruing to *FIRMS*
$\overline{\omega_{LDg}}$	Share of land rent accruing to *GOV*
$\overline{\omega_{OTh}}$	Share of others transfers accruing to *HH*
$\overline{\omega_{OTf}}$	Share of others transfers accruing to *FIRMS*
$\overline{\omega_{OTg}}$	Share of others transfers accruing to *GOV*
$\overline{\tau_{hf}}$	Rate of indirect tax levied on *HH* and accruing to *FIRMS*
$\overline{\tau_{hg}}$	Rate of indirect tax levied on *HH* and accruing to *GOV*
$\overline{\tau_{Ih}}$	Rate of tax on income of *HH*
$\overline{\tau_{If}}$	Rate of corporation tax (on *FIRMS*)
$\overline{\alpha_i}$	Share coefficient for good *i* in LES utility function
$\overline{\lambda_{ij}}$, $\overline{\lambda_{Lj}}$, $\overline{\lambda_{Kj}}$, $\overline{\lambda_{LDj}}$	Share coefficients of CES production function for intermediate input *i*, labor, capital and land for production of sector *j*
$\overline{\beta_i}$	coefficient linking total fixed capital consumption and fixed capital formation of good *i*
$\overline{\gamma_{ij}}$	CO2 emission factor linked to consumption of good *i* in production of good *j*
$\overline{\gamma_{ih}}$	CO2 emission factor linked to consumption of good *i* in final consumption by *HH*

Table 5: List of calibrated parameters

Table 7 shows the values of a first set of structural parameters in the reference setting of Imaclim-BR 6.

Basic needs and *sh* shares parameters are estimated to reflect slightly contrasted income elasticities of consumed goods and services. Subsequently, energy goods, food and manufactured goods and services show respective 1.1, 0.9 and 1 income elasticities.

In production trade-offs the elasticities of substitution between the varying part of technical coefficients are set to $\sigma = 1.2$ for all sectors. Similarly as the LES demand system, technical asymptotes makes it possible to adjust output and furthermore substitution elasticities of the different inputs and factors. For some sectors like the power sector, the choice of technical asymptotes is made to mimic bottom-up scenarios about the abatement potential of the power sector until 2030. In practice, following the national energy plan, the power generation system would not change significantly with a carbon price of 100$US: around 5-10 Mt of emissions abatement would result from additional hydro power and wind power capacities. Table 8 shows the values of technical asymptotes for all production factors and sectors.

As shown in table 9, the elasticity of the wage curve of -0.3 is taken from Estevão and Carvalho Filho (2012) and reflects rather flexible wages. Finally, with $sh_r = 0.5$, the wage curve is half indexed on nominal and half indexed on real wage in the reference setting.

A.2.2 Specific non-calibrated parameters in Imaclim-BR 12-ext reference setting

Table 10 shows the set of specific non-calibrated parameters of Imaclim-BR 12-ext version.

Table 11 shows the values of the specific structural parameters.

Non-calibrated parameters	Description
	Structural parameters
sh_i	Share of base year consumption per capita needed to compute basic needs of good i
σ_j	Elasticity of substitution between production factors of sector j
β_{ij}, β_{Lj}, β_{Kj}, β_{LDj}	Shares of base year technical coefficients corresponding to floor values for intensity of input i, labor, capital and land for production of sector j
σ_{Mi}	Elasticity of the ratio of imports to domestic production of energy good i, to the corresponding terms of trade
σ_{MYi}	Elasticity of imports of non-energy good i to GDP - "income" elasticity of imports
σ_{Xi}	Elasticity of exports of good i to the corresponding terms of trade
σ_u	Elasticity of efficient wage to unemployment rate
sh_r	level of indexation of the wage curve on real wages
	Scenarios parameters
δ_N	Rate of total population growth
δ_{NS}	Rate of active population growth
ϕ_{Lj}	Rate of labor productivity gain of sector j implementing as factor augmenting
u_{ref}	Reference rate of unemployment
ψ_{ij}	Rate of decrease of production intensity of input i for production of good j - structural driver
ψ_{Kj}	Rate of decrease of capital intensity of production of good j - structural driver
ψ_{LDj}	Rate of decrease of land intensity of production of good j - structural driver
$\delta_{p_{Mi}}$	Rate of variation of world price of good i compared to composite
τ_S	Saving rate of HH
sh_{Mi}	exogenous variation of M/Y ratio compared to base year outside terms of trade for energy sector i - non-price structural change of energy imports
δ_{Xi}	Rate of growth of exports outside terms of trade - structural change for energy sectors and proxy of rest of the world growth for non-energy sectors
t_{Carb}	Carbon tax
t	time horizon

Table 6: List of non-calibrated parameters

Parameters	Values					
	BIO	FF	ELEC	AGRI	INDUS	COMP
sh	0	0	0	0.2	0.1	0.1
σ	1.2	1.2	1.2	1.2	1.2	1.2
σ_M	1.2	1.2	1.2	1.2	1.2	1.2
σ_{MYi}	-	-	-	1.1	1.1	1.1
σ_X	1	1	1	1	1	1

Table 7: values of structural parameters

Parameters	Values					
	BIO	FF	ELEC	AGRI	INDUS	COMP
	0.8	0	0.9	0	0	0
	0	0.8	0.9	0	0	0
	0.5	0.5	0.9	0	0	0
	0.95	0.95	0.95	0.95	0.95	0.75
	0.95	0.95	0.95	0.95	0.95	0.75
	0.95	0.95	0.95	0.95	0.95	0.75
β_L	0.8	0.8	0.8	0.8	0.8	0.75
β_K	0.8	0.8	0.8	0.8	0.8	0.75
β_{LD}	0.8	0.8	0.8	0.8	0.8	0.75

Table 8: values of structural parameters

Parameters	Values
σ_u	-0.3
sh_r	0.5

Table 9: values of structural parameters

Parameter	Description
	Structural parameters
sh_i	Share of base year consumption per capita needed to compute basic needs of good or service *i*
σ_{MR}	Elasticity of substitution between food goods
σ_{RES}	Elasticity of substitution between housing energy sources in final demand
σ_{VF}	Elasticity of substitution between *INDVHC* and *FTRA* in final demand
σ_F	Elasticity of substitution between *BIOTRA* and *ROILTRA*
	Scenarios parameters
δ_{RESi}	Rate of non-price based change of intensity of energy source *i* in residential energy demand
$\delta_{EffFuel}$	Rate of fuel efficiency gain of private transportation - AEEI
δ_{YOIL}	Rate of increase of total domestic oil production

Table 10: Specific non-calibrated parameters

Parameters	Values			
	FD	TRA	RES	OTH
sh_i	0.15	0	0.1	0.1
σ_{MR}	0.1			
σ_{RES}	0.4 (Gurgel and Paltsev, 2014)			
σ_{VF}	0.3 (Gurgel and Paltsev, 2014)			
σ_F	5			

Table 11: values of specific structural parameters

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Working Paper
N°2016-05-23